Competency Based Practice Questions

Mathematics - XII

Co-created by

Board of School Education Haryana
and
Educational Initiatives

HOW TO USE THIS BOOKLET

Dear Teachers and Students,

The **Board of School Education Haryana** is pleased to present the **Competency-Based Practice Questions** booklet. This resource has been thoughtfully designed to help you deepen your understanding of key concepts and enhance your problem-solving skills. It includes **50 exemplar questions** carefully aligned with the curriculum to familiarize students with the format of **Competency-Based Questions**. These questions are intended to support targeted practice and develop the skills necessary to confidently approach a variety of question types in assessments.

Best Ways for Teachers to Utilise This Resource

1. Integrate into Classroom Teaching

- Use these questions to demonstrate how theoretical concepts translate into practical applications.
- Encourage group discussions to explore reasoning and understanding of concepts taught.

2. Scaffold Student Learning

- Start with simpler questions and guide students through the thought process.
- Gradually introduce more complex questions to build confidence and familiarity.

3. Incorporate into Assessments

- Use these questions in classroom quizzes or homework to help students adapt to the format.
- Provide feedback that emphasises reasoning over correctness, encouraging students to refine their understanding.

4. Focus on Skill Development

- Highlight how these questions nurture understanding, analysis and critical thinking.
- Use student responses to identify and address misconceptions effectively.

Best Ways for Students and Parents to Utilise This Resource

1. Focus on Conceptual Understanding

- Approach each question as a way to understand *why* and *how* a concept works, rather than simply finding the correct answer.

2. Practice Purposefully

- Don’t rush—break down the question, identify the concept it addresses, and plan your approach before solving it.

3. Use Feedback to Improve

- Treat mistakes as learning opportunities. Review incorrect answers to understand *what went wrong* and *how to improve*.
- Revisit similar questions to build confidence and mastery over the topic.

Best Ways for Parents to Utilise This Resource

1. Encourage Critical Thinking

- Spend time discussing questions and concepts, asking “Why?” and “How?”.

2. Create a Positive Environment

- Celebrate effort and curiosity, not just grades.
- Help your child view mistakes as opportunities to learn and grow.

3.Collaborate with Teachers

- Stay informed about competency-based assessments through school communications.
- Share observations and work with teachers to address any concerns or challenges.

Final Message

These practice questions are an excellent opportunity to strengthen your conceptual understanding and boost your confidence in solving competency-based questions. For students, each question builds skills that will help you tackle similar challenges with ease. For teachers, this is a chance to mentor students in developing their thinking and problem-solving skills.

Start today—every effort you invest will prepare you not only for exams but for a lifetime of meaningful learning and success. Let’s make this journey toward competency-based education a meaningful and successful one!

Board of School Education, Haryana

Contents

Relations and Functions

Serial No.	Question	Marks
1	Which of the following is NOT an equivalence relation on the set Q = $\{w, x, y, z\}$? (A) $R_1 = \{(w, w), (x, x), (y, y), (z, z)\}$ (B) $R_2 = \{(w, x), (x, w), (w, w), (x, x), (y, y), (z, z)\}$ (C) $R_3 = \{(w, w), (x, x), (y, y), (z, z), (w, x), (x, w), (y, x), (x, y)\}$ (D) $R_4 = \{(w, w), (x, x), (y, y), (z, z), (w, z), (z, w), (x, y), (y, x)\}$ [Skill: Mechanical]	1
2	X and Y are finite sets such that it is possible to define a one-one function from X to Y. Based on the above information, two statements are given below - one labelled Assertion (A) and the other labelled Reason (R). Read the statements carefully and choose the option that correctly describes statements (A) and (R). *Assertion (A)*: It is always possible to define an onto function from set X to set Y. *Reason (R)*: $n(\mathrm{X}) \leq n(\mathrm{Y})$ (A) Both (A) and (R) are true and (R) is the correct explanation for (A). (B) Both (A) and (R) are true but (R) is not the correct explanation for (A). (C) (A) is true but (R) is false. (D) (A) is false but (R) is true. [Skill: Understanding]	1
3	Two functions $f : R \rightarrow R$ and $g : R^+ \rightarrow R$ are such that: $f(x) = e^x - 1 \ and \ f(g(x)) = x$ Find $g(2)$. [Skill: Understanding]	3

Marking Scheme

Q No.	Rubric	Marks
1	**Correct Answer:** C R_3 is not transitive as (w, x) & $(x, y) \in R_3$, but $(w, y) \notin R_3$.	1
	A: Students selecting this option might have failed to either identify this as symmetric or as transitive.	
	B: Students selecting this option might have failed to identify this as transitive.	
	D: Students selecting this option might have failed to identify this as transitive.	
2	**Correct Answer:** D Given that it is possible to define a one-one function from X to Y, which means that $n(X) \leq n(Y)$. Therefore, reason is true. But if $n(X) < n(Y)$, then it is not possible to define an onto function from X to Y. Therefore, assertion is false.	1
	A: Students selecting this option may think that if a one-one function can be defined from X to Y, then an onto function can also be defined from X to Y, the reason being $n(X) \leq n(Y)$.	
	B: Students selecting this option may think that if a one-one function can be defined from X to Y, then an onto function can also be defined from X to Y, but $n(X) \leq n(Y)$ is not the reason for it.	
	C: Students selecting this option understand when an onto function can be defined, but do not know that existence of a one-one function form X to Y implies $n(X) \leq n(Y)$.	
3	**Evaluation Criteria:** Uses the given information $f\big(g(x)\big) = x$ to write: $e^{g(x)} - 1 = x$	1
	Simplifies the above to get: $e^{g(x)} = x + 1$ $\Rightarrow g(x) = ln(x + 1)$	1
	Finds $g(2)$ as $\ln(2 + 1) = \ln 3$. (Award full marks if log is used instead of ln.)	1

4	In a basketball game, the angle θ (in radians) of a player's throw is related to the height h the ball reaches by the equation: $\theta = 2sin^{-1}\frac{h}{4}$ If the ball reached a height of 2 metres, at what angle was the ball thrown? (A) $\frac{\pi}{6}$ radians (B) $\frac{\pi}{4}$ radians (C) $\frac{\pi}{3}$ radians (D) $\frac{\pi}{2}$ radians [Skill: Application]	1
5	What is the range of the function given below? $f(x) = 6sin^{-1}(\frac{x^2}{x^2+1})$ (A) [0, 3π) (B) [-3π, 3π] (C) [0, $\frac{\pi}{2}$) (D) $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ [Skill: Understanding]	1
6	(i) Given: $\theta = 2\sin^{-1}\left(\frac{1}{3}\right)$ Find the value of $\cos\theta$. (ii) Express the below expression in the simplest form. $\tan^{-1}\left(\frac{\sin x}{1+\cos x}\right)$ [Skill: Mechanical]	5

Marking Scheme

Q No.	Rubric	Marks
4	**Correct Answer:** C At 3 metres height, $\theta = 2\sin^{-1}(2/4)$ $\theta = 2\sin^{-1}(1/2)$ Since $\sin^{-1}(1/2) = \pi/6$ $\theta = 2 \times \pi/6 = \pi/3$.	1
	A: Students selecting this option only find that $\sin^{-1}(1/2) = \pi/6$ and do not multiply this with 2.	
	B: Students selecting this option think that $\sin^{-1} 1/2 = \pi/8$.	
	D: Students selecting this option think that $\sin^{-1}(1/2) = \pi/4$.	
5	**Correct Answer:** A $\text{Sin}^{-1}(z) \in [-\pi/2, \pi/2]$. Since $x^2/(x^2 + 1)$ lies in $[0, 1)$, $\sin^{-1}(x^2/(x^2 + 1)) \in [0, \pi/2)$. Multiplying range $[0, \pi/2)$ by 6 to get $[0, 3\pi)$.	1
	B: Students selecting this option have multiplied 6 with $\text{Sin}^{-1}(z) \in [-\pi/2, \pi/2]$.	
	C: Students selecting this option have not multiplied the range $[0, \pi/2)$ by 6.	
	D: Students selecting this option have found the range of $\sin^{-1}(1/2)$.	
6	**Evaluation Criteria:** (i) Writes that $sin\left(\frac{\theta}{2}\right) = \frac{1}{3}$	0.5
	Using the formula $cosx = 1 - 2sin^2\left(\frac{x}{2}\right)$, writes that, $cos\theta = 1 - 2 \times \frac{1}{9} = \frac{7}{9}$.	1.5
	(ii) Writes that, $\frac{\sin x}{1 + \cos x} = \frac{2\sin\frac{x}{2}\cos\frac{x}{2}}{2 - 2\sin^2\frac{x}{2}}$	1
	Simplifies the above equation to $\tan\frac{x}{2}$.	1
	Writes that $tan^{-1}1\left(tan\frac{x}{2}\right) = \frac{x}{2}$.	1

Matrices

7	Let X and Y be 3×3 matrices such that X is symmetric and Y is skew-symmetric. Consider the matrix Z = X + Y. Which of the following statements is always true about Z? (A) Z + Z' = 2(X + Y) (B) Z + Z' = 2X (C) Z' = Z (D) Z' = -Z [Skill: Mechanical]	1
8	A = $\begin{bmatrix} \sin^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \cos^2\theta \end{bmatrix}$ and B = $\begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$ (i) Find one value of θ for which A = B. (ii) Find matrices A and B for this value of θ. [Skill: Application]	2
9	Let A and B be invertible 2×2 matrices such that A $\neq$ B and A + B = AB. Prove that $A^{-1} + B^{-1} = I$, where I is the identity matrix. [Skill: Application]	3
10	Let A be a 3×3 matrix such that A + A' = 2I, where I is the identity matrix. Express matrix A as the sum of S and K, where S is a symmetric matrix and K is a skew-symmetric matrix, both independent of A'. [Skill: Understanding]	3

Marking Scheme

Q No.	Rubric	Marks
7	**Correct Answer:** B X' = X Y' = -Y Z = X + Y Z' = (X + Y)' = X' + Y' = X - Y Z + Z' = X + Y + X - Y = 2X	1
	A: Students selecting this option think that Y' = Y.	
	C: Students selecting this option think that Z is symmetric.	
	D: Students selecting this option think that Z is skew-symmetric.	
8	**Evaluation Criteria:** (i) Writes that as A = B, equating the first element in the first row gives: $sin^2\theta = cos^2\theta$.	0.5
	Simplifies the above equation to find any one value of θ as: $sin^2\theta = 1 - sin^2$ $\Rightarrow sin^2\theta = \frac{1}{2}$ $\Rightarrow\ \theta =\ 45°$	0.5
	(ii) Writes that for $\theta = 45°, sin45° = \frac{1}{\sqrt{2}} and\ cos45° = \frac{1}{\sqrt{2}}$.	0.5
	Forms the matrices A and B as follows, $A = \begin{bmatrix} \left(\frac{1}{\sqrt{2}}\right)^2 & \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} & \left(\frac{1}{\sqrt{2}}\right)^2 \end{bmatrix}$ and $B = \begin{bmatrix} \left(\frac{1}{\sqrt{2}}\right)^2 & \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}} & \left(\frac{1}{\sqrt{2}}\right)^2 \end{bmatrix}$ Concludes that: $A = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$ and $B = \begin{bmatrix} \frac{1}{2} & \frac{1}{2} \\ \frac{1}{2} & \frac{1}{2} \end{bmatrix}$	0.5
9	**Evaluation Criteria:** Multiplies both sides of A + B = AB: on the left by A^{-1} and on the right by B^{-1} as: $A^{-1}(A + B)B^{-1} = A^{-1}(AB)B^{-1}$	1

	Simplifies LHS as follows, $A^{-1}(A+B)B^{-1} = A^{-1}.A.B^{-1} + A^{-1}.B.B^{-1}$ $= I \times B^{-1} + A^{-1} \times I$ $= B^{-1} + A^{-1}$	1
	Simplifies RHS as follows, $A^{-1}(AB)B^{-1} = (A^{-1} \times A)(B \times B^{-1}) = I$ Hence proves that $A^{-1} + B^{-1} = I$.	1
10	**Evaluation Criteria:** Identifies that $S = \frac{A+A'}{2}$ and $K = \frac{A-A'}{2}$	1
	Substitutes A + A' = 2I into S as follows, $S = \frac{A+A'}{2} = \frac{2I}{2} = I$ Concludes that identity matrix (S) is symmetric. (Award full marks for any other correct method.)	0.5
	Rearranges A + A' = 2I to get A' = 2I - A Substitutes the above into K as follows, $K = \frac{A-(2I-A)}{2} = A - I$ Writes that, (A - I)' = A' - I' = 2I - A - I = I - A = -(A - I) Hence K is skew symmetric.	1
	Writes that, A = I + (A - I). Hence A = S + K where S is symmetric and K is skew symmetric.	0.5

Determinants

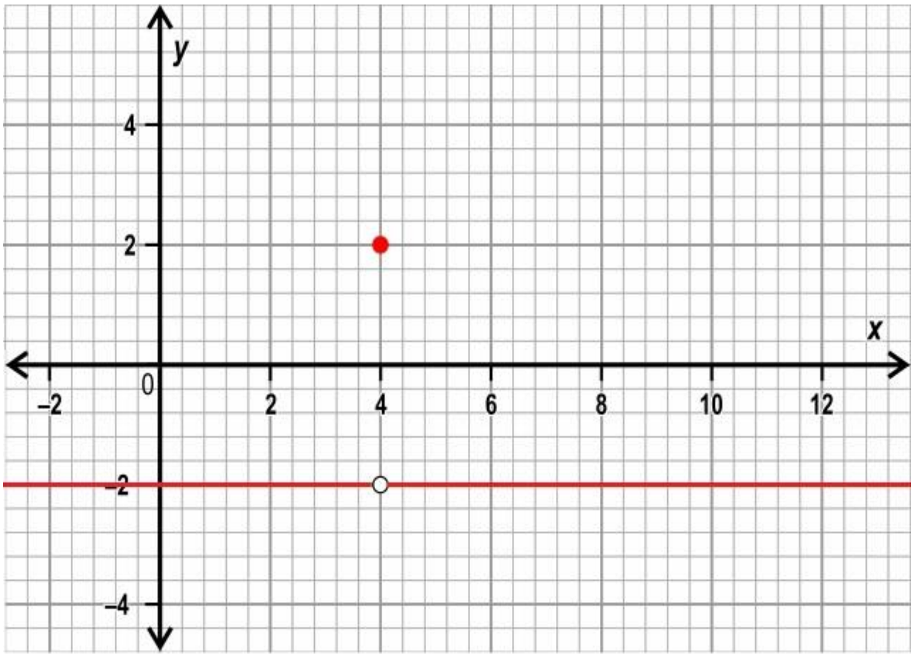
11	Given the system of equations: $2x + y = 5$ $x + 3y = 7$ This system can be written in matrix form as PX = Q, where P is the coefficient matrix, X is the variable matrix, and Q is the constant matrix. Which of the following statements is correct? (A) The system has no solution because P is a singular matrix. (B) The system has a unique solution because P is a non-singular matrix. (C) The system has infinitely many solutions because P is a singular matrix. (D) The system has infinitely many solutions because P is a non-singular matrix. [Skill: Understanding]	1
12	A and B are 3×3 matrices such that $\|A\| = -4$ and $B = 2A$. If adjoint B = $\begin{bmatrix} 16 & 0 & 0 \\ 0 & 16 & 0 \\ 0 & 0 & 16 \end{bmatrix}$, find B^{-1}. [Skill: Mechanical]	3
13	Three points P(1, k), Q(4, 2), R(5, k + 1) are in a coordinate plane. (i) Determine the value(s) of k for which these points are collinear. (ii) If k = 3, find the area of the triangle formed by points P, Q and R. [Skill: Understanding]	5

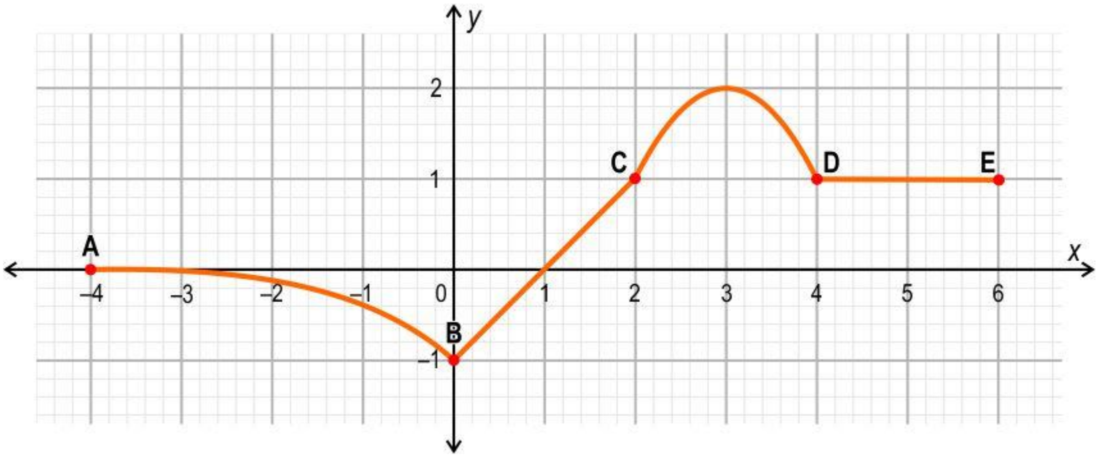
Marking Scheme

Q No.	Rubric	Marks
11	**Correct Answer:** B Since $det(P) = 5 \neq 0$, matrix P is non-singular. Hence, the system of equations has a unique solution.	1
	A: Students selecting this option do not know that P is a non-singular matrix since $det(P) \neq 0$.	
	C: Students selecting this option do not know that P is a non-singular matrix since $det(P) \neq 0$.	
	D: Students selecting this option do not know that a non-singular matrix lead to a unique solution.	
12	**Evaluation Criteria:** Writes that $\lvert B\rvert = \lvert 2A\rvert$.	0.5
	Writes that $\lvert kA\rvert = k^n\lvert A\rvert$ Identifies that $k = 2$ and $n = 3$ Hence, $\lvert 2A\rvert = 2^3\lvert A\rvert$	1
	Writes that |B| = 8|A| = 8 \times (-4) = -32.	0.5
	Writes that $B^{-1} = \frac{adjB}{\lvert B\rvert}$ Hence $B = \begin{bmatrix} -\frac{1}{2} & 0 & 0 \\ 0 & -\frac{1}{2} & 0 \\ 0 & 0 & -\frac{1}{2} \end{bmatrix}$	1
13	**Evaluation Criteria:** (i) Writes that if the points are collinear, then, $\begin{vmatrix} 1 & k & 1 \\ 4 & 2 & 1 \\ 5 & k+1 & 1 \end{vmatrix} = 0$	1
	Simplifies the above determinant as follows: $1[2 - (k+1)] + 4(k + 1 - k) + 5(k - 2) = 0$ $4k - 5 = 0$ $k = \frac{5}{4}$	1.5
	(ii) Writes that P = (1, 3), Q = (4, 2), R = (5, 4) Writes that area of triangle $= \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 4 & 2 & 1 \\ 5 & 4 & 1 \end{vmatrix}$	1

	Calculates the area of triangle as follows, $\frac{1}{2}[1(2-4)+4(4-3)+5(3-2)]=\frac{7}{2}$ Concludes that the area of triangle $=\frac{7}{2}=3.5$ sq units.	1.5

Continuity and differentiability

14	Shown below is the graph of a function $y = f(x)$.  Which of the following is true about $y = f(x)$? (A) $\lim_{x\to 4} f(x)$ does not exist (B) $\lim_{x\to 4} f(x)$ exists but $f(x)$ is not continuous at $x = 4$ (C) $\lim_{x\to 4} f(x)$ exists and $f(x)$ is continuous at $x = 4$ (D) (Such a function cannot exist.) [Skill: Understanding]	1
15	Find $\frac{dy}{dx}$ where, $y = \left(e^{\operatorname{cosec} x} + x\right)^2$ [Skill: Mechanical]	2
16	If $x = k\,(\log t)$ and $y = k\left(\tan^{-1} t\right)$, where k is a constant, find $\frac{dy}{dx}$. [Skill: Mechanical]	2

17	Shown below is the graph of a function $f(x)$. The graph is divided into 4 sections - AB, BC, CD, and DE.  Analyse the graph and determine in which of the four section(s), is the second derivative of the function, $f''(x) = 0$. Give valid reasons. [Skill: Understanding]	3

Marking Scheme

Q No.	Rubric	Marks
14	**Correct Answer:** B From the graph, it can be seen that f(x) approaches (-2) as x approaches 4 on both sides. Hence, $\lim_{x \to 4} f(x)$ exists. But $\lim_{x \to 4} f(x) \neq f(4)$, thus f(x) is not continuous at x = 4.	1
	A: Students selecting this option think that for a limit to exist at a point c, LHL=RHL=f(c)	
	C: Students selecting this option do not understand the meaning of continuity.	
	D: Students selecting this option lack understanding of functions and continuity.	
15	**Evaluation Criteria:** Differentiates *y* with respect to *x* using chain rule as: $\frac{dy}{dx} = 2(e^{cosecx} + x) \times \frac{d}{dx}(e^{cosecx} + x)$	1
	Simplifies the above differential as: $\frac{dy}{dx} = 2(e^{cosecx} + x)(-e^{cosecx} cosecx \cot x + 1)$	1
16	**Evaluation Criteria:** Finds the derivative of *x* with respect to *t* as $\frac{dx}{dt} = \frac{k}{t}$.	0.5
	Finds the derivative of *y* with respect to *t* as $\frac{dy}{dt} = \frac{k}{(1+t^2)}$.	0.5
	Finds $\frac{dy}{dx}$ as: $\frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{t}{(1+t^2)}$.	1
17	**Evaluation Criteria:** Writes that in section BC, $f''(x) = 0$.	0.5
	Gives a reason. For example, the slope of the function in the section BC is a constant, which means $f^{(x)} = k$, where k is a constant. Hence, $f''(x) = 0$in BC.	1
	Writes that in section DE, $f''(x) = 0$.	0.5
	Gives a reason. For example, the function in the section DE is a constant, which means $f(x) = k$, where k is a constant. Hence, $f'(x) = 0 = f''(x)$ in DE.	1

Application of derivatives

18	A potter is making a right circular cylindrical pot where the radius and height are always equal. The radius of the pot is increasing at a rate of 2 cm/s.  *(Note: The image is for representation purpose only and is not to scale.)* Which of the following is the rate of change of volume of the cylindrical pot, in terms of π, when its radius is 5 cm? (A) 40π cm^3/s (B) 75π cm^3/s (C) 150π cm^3/s (D) 200π cm^3/s [Skill: Application]	1
19	A hardware engineer measured her smartphone's battery temperature (in °C) during the daily usage and modelled it by the function $T(t) = t^3 - 6t^2 + 9t + 20$, where $t \in [0, 5]$ represents time in hours. Determine the time when the battery's temperature is maximum and minimum in the duration of 5 hours. [Skill: Application]	3
20	A cement manufacturing company wants to analyse its profit based on the quantity of cement sold. The profit function $P(x)$ (in lakhs of rupees) is given by: $P(x) = x\ log(kx), x > 0, k > 0$,where x represents the quantity of cement sold (in tons) and k is a scaling factor The company wants to identify the intervals where increasing sales lead to increased profit and where further sales may result in decreasing profit. Determine these intervals. [Skill: Understanding]	2

21	A manufacturer produces right conical tents with a cloth that covers the curved surface area of $9\sqrt{3}\pi$ m^2. Determine the dimensions (radius and height) that will maximise the volume of the tent, with $r < 4$ m. *(Note: The curved surface area of a right cone is $\pi r l$ where l is the slant height of the cone.)* [Skill: Application]	5

Marking Scheme

Q No.	Rubric	Marks
18	**Correct Answer:** C Volume of cylinder V = $\pi r^2 h$ Given that r = h at any point of time Thus, V = πr^3 => $dV/dt = 3\pi r^2\, dr/dt = 3\pi \times 25 \times 2 = 150\pi$	1
	A: Students selecting this option may have used the formula for surface area of cylinder.	
	B: Students selecting this option may have used the correct formula $\pi r^2 h$ for volume of cylinder but did not multiply with dr/dt on differentiating.	
	D: Students selecting this option may have used the formula of volume of sphere instead of cylinder.	
19	**Evaluation Criteria:** Differentiates the temperature function as: $T'(t) = (t^3 - 6t^2 + 9t + 20) = \frac{d}{dt}(t^3) - \frac{d}{dt}(6t^2) + \frac{d}{dt}(9t) + \frac{d}{dt}(20)$ $\Rightarrow T^{'}(t) = 3t^2 - 12t + 9$	0.5
	Finds the critical points by solving for *t* when T'(*t*) = 0 as: $3t^2 - 12t + 9 = 0$ $\Rightarrow 3(t^2 - 4t + 3) = 0$ $\Rightarrow 3(t-3)(t-1) = 0$ Concludes that *t* = 1 hour and *t* = 3 hours are the critical points.	1
	Finds the second derivative of T(*t*) as: $T''^{(t)} = 6t - 12$	0.5
	Determines the nature of critical points as: At *t* = 1: $T''(1) = 6 - 12 = -6 < 0$ Hence, *t* = 1 hour is the point of local maximum where the temperature is maximum. $(T''(1) = 24°C)$ At *t* = 3: $T''(t) = 6 \cdot 3 - 12 = 6 > 0$ Hence, *t* = 3 hour is the point of local minimum where the temperature is minimum. $(T(3) = 20°C)$	1

20	**Evaluation Criteria:** Differentiates $P(x)$: $P'(x) = \frac{d}{dx}(x\log(kx)) = \log(kx)\cdot\frac{d}{dx}(x) + x\cdot\frac{d}{dx}\cdot(\log(kx))$ $\Rightarrow P'(x) = \log(kx) + x\cdot\frac{1}{x} = \log(kx) + 1$	1
	Analyses the sign of $P'(x)$: From $P'(x) = \log(kx) + 1$: $P'(x) > 0\ when\ \log(kx) \succ 1$, which simplifies to $kx > e^{-1}$ or $x > \frac{1}{ke}$ $P'(x) < 0$ when $\log(kx) < -1$, or $x < \frac{1}{ke}$. Concludes that the profit function is: 1. Increasing for $\frac{1}{ke} < x < \infty$ 2. Decreasing for $0 < x < \frac{1}{ke}$	1
21	**Evaluation Criteria:** Writes that $l = \sqrt{r^2 + h^2}$, where h is the (vertical) height of the conical tent. Derives the relationship between h and r as: $\pi rl = 9\sqrt{3}\pi$ $\Rightarrow r\sqrt{r^2+h^2} = 9\sqrt{3}$ $\Rightarrow r^2 + h^2 = \left(\frac{9\sqrt{3}}{r}\right)^2$ $\Rightarrow h = \sqrt{\frac{243}{r^2} - r^2}$	1
	Expresses the volume function in terms of r as: $V = \frac{1}{3}\pi r^2 h = \frac{1}{3}\pi r^2\sqrt{\frac{243}{r^2} - r^2}$ $\Rightarrow V = \frac{1}{3}\pi\sqrt{243r^2 - r^6}$	1
	Finds the critical point by differentiating the volume function with respect to r as: $\frac{dV}{dr} = \frac{1}{3}\pi\frac{d}{dr}\left(\sqrt{243r^2 - r^6}\right)$	1.5

	$= \frac{1}{3}\pi \frac{2 \times 243r - 6r^5}{2\sqrt{243r^2 - r^6}}$ $= \frac{1}{3}\pi \frac{243 - 3r^4}{\sqrt{243 - r^4}}$ Equates the first derivative to 0: $\pi \frac{81 - r^4}{\sqrt{243 - r^4}} = 0$ $\Rightarrow 81 - r^4 = 0$ $\Rightarrow r = 3$	
	States that: At $r < 3: r^4 < 81 \Rightarrow \frac{dV}{dr} > 0$: The volume function is increasing for $r < 3$. At $r > 3: r^4 > 81 \Rightarrow \frac{dV}{dr} < 0$: The volume function is decreasing for $r > 3$. Hence concludes that the volume function maximises at $r = 3$ m. (Award full marks if second derivative test is used instead of first derivative test.)	1
	Finds the value of h as: $h = \sqrt{\frac{243}{9} - 9} = \sqrt{27 - 9} = \sqrt{18} = 3\sqrt{2}m$	0.5

Integrals

22	Shown below is an integral where *k* is a constant. $\int \frac{1}{t^2-k^2} d(t) = g(t) + C$ Which expression correctly represents the following integral? $\int \frac{1}{k^2-t^2} d(t)$ (A) -*g*(*t*) + C (B) *g*(*t*) + C (C) $\frac{1}{t} tan^{-1}\frac{k}{t} + C$ (D) $log\|k+\sqrt{k^2-t^2}\| + C$ [Skill: Understanding]	1
23	In the equation shown below, C is an arbitrary constant. $\int x \sec^2(x)\,dx = h(x) + \log\|\cos x\| + C$ Find *h*(*x*). [Skill: Mechanical]	2
24	$I = \int_{-25}^{25} \left(3x^3 + \sqrt{5}x\right) dx$ Which of the following statements is(are) correct? Justify your answer. i) The function being integrated is an even function. ii) Value of this integral is 0. [Skill: Understanding]	2
25	The area bounded by the curve $y = x^2 + \sin x$ and the *x*-axis from $x = 0$ to $x = k$ is given by the area function: A(*k*) = $\int_0^k \left(x^2 + \sin x\right) dx$ Find the value of $A\left(\frac{\pi}{2}\right)$. [Skill: Application]	3

26	In the following equation, C is a constant of integration, and *h*(*x*) and *k*(*x*) are functions. $\int \frac{x^3}{x^4 - 5x^2 + 6} dx = -h(x) + \frac{3}{2} k(x) + C$ (i) Find the value of the functions *h*(*x*) and *k*(*x*). (ii) Determine the value of *h*($\sqrt{2}$). [Skill: Understanding]	5

Marking Scheme

Q No.	Rubric	Marks
22	**Correct Answer:** A We know that, $$\int \frac{dx}{a^2 - x^2} = \frac{1}{2a} log\|\frac{x+a}{x-a}\| + C$$ We also know that, $$\int \frac{dx}{x^2 - a^2} = \frac{1}{2a} log\|\frac{x-a}{x+a}\| + C$$ Hence, $\int \frac{1}{k^2 - t^2} d(t)$ = - $\int \frac{1}{t^2 - k^2} d(t)$	1
	B: Students selecting this option do not consider the negative sign.	
	C: Students selecting this option have considered the identity, $\int \frac{dx}{x^2 + a^2}$.	
	D: Students selecting this option have considered the identity, $\int \frac{dx}{\sqrt{x^2 - a^2}}$.	
23	**Evaluation Criteria:** Integrates using integration by parts as follows: $$\int xsec^2(x)dx = x\tan(x) - \int \tan(x)\, dx$$	1
	Solves the above equation as follows: $$\int xsec^2(x)dx = x\tan(x) + \log\|\cos(x)\| + C$$	0.5
	Compares the above equation with the given equation to find $h(x)$ as $x \tan(x)$.	0.5
24	**Evaluation Criteria:** i) Writes that first statement is incorrect as here, $f(-x) = -f(x)$ for all x. Hence it is an odd function, not even.	1
	ii) Writes that the second statement is correct as integral of odd function is 0.	1
25	**Evaluation Criteria:** Sets up $A\left(\frac{\pi}{2}\right) = \int_0^{\frac{\pi}{2}} (x^2 + \sin x)dx$ and integrates as follows: $$\int_0^{\frac{\pi}{2}} x^2\, dx = \frac{\pi^3}{24}$$	2

	$\int_0^{\frac{\pi}{2}} (\sin x) dx = 1$	
	Combines results to obtain $A\left(\frac{\pi}{2}\right) = \frac{\pi^3}{24} + 1$ sq units.	1
26	**Evaluation Criteria:** (i) Substitutes $t = x^2$ and $dx = \frac{dt}{2x}$, and rewrites the integral as $\frac{1}{2}\int \frac{t}{t^2-5t+6} dt$.	1
	Factors the expression $t^2 - 5t + 6\ as\ (t-2)(t-3)$. Sets up the partial fraction decomposition as $\frac{t}{(t-2)(t-3)} = \frac{A}{t-2} + \frac{B}{t-3}$.	1
	Sets up and solves the system of equations for A and B, obtaining A = −2 and B = 3.	0.5
	Substitutes A = -2 and B = 3 into the integral and rewrites the expression as, $\frac{1}{2}\left(-2\int \frac{1}{t-2} dt + 3\int \frac{1}{t-3} dt\right)$ Correctly integrates each term to find, $-\log\lvert t-2\rvert + \frac{3}{2}\log\lvert t-3\rvert + C$	1
	Substitutes $t = x^2$ to get, $-\log\lvert x^2-2\rvert + \frac{3}{2}\log\lvert x^2-3\rvert + C.$ Hence finds, $h(x) = \log\lvert x^2-2\rvert$ $k(x) = \log\lvert x^2-3\rvert$	1
	(ii) Finds $h(\sqrt{2})$ as, $\log\left\lvert(\sqrt{2})^2 - 2\right\rvert = \log(0)$ Writes that this is undefined.	0.5

Application of integrals

27	Which of the following definite integrals represent the area bounded by $y = x^2$, $x = -2$, $x = 2$, and the x-axis? (i) $\int_0^2 x^2\,dx$ (ii) $\int_{-2}^2 x^2\,dx$ (iii) $2 \times \int_0^2 x^2\,dx$ (A) only (ii) (B) only (iii) (C) both (i) and (ii) (D) both (ii) and (iii) [Skill: Understanding]	1
28	A local bicycle-sharing service tracks the number of bicycle rentals throughout the day. The rate of bicycle rentals (in bicycles per hour) is modelled by the function: $f(x) = 3x^2 + 2x + 1$ where x represents the hours since the start of the bicycle-sharing service at 6:00 AM. Calculate the total number of bicycles rented between 6:00 AM and 9:00 AM ($0 \le x \le 3$). [Skill: Application]	2
29	An environmental research team is studying the pollution concentration in a river. The pollution concentration (in ppm or parts per million) varies along the river's length and is described by the function: $c(x) = 1 + 2x - \frac{x^2}{3}$ where x represents the distance (in kilometres) from the river's source. Calculate the average pollution concentration between $x = 0$ and $x = 6$ kilometres. [Skill: Application]	3

Marking Scheme

Q No.	Rubric	Marks
27	**Correct Answer:** D The points of intersection between the given curves are (-2,4) and (2,4). Based on this, (ii) is the direct interpretation of the definite integral of the area enclosed between the two curves. Since the function x^2 is an even symmetrical function about y-axis, the area enclosed is also symmetrical about the same axis. Hence, option (iii) is also correct interpretation.	1
	A: Students selecting this option might have correctly identified the points of intersection, but failed to realise that the function is even and hence symmetrical about y-axis.	
	B: Students selecting this option might have only focussed on the symmetrical nature of the function, and ignored the (ii) option which is also correct.	
	C: Students selecting this option might have incorrectly assumed that the lower bound must always be 0, and ignored the symmetrical nature of the function.	
28	**Evaluation Criteria:** Sets up the integral as: $\int_0^3 (3x^2 + 2x + 1)\ dx$	0.5
	Integrates each term in the expression 3x^2+2x+1 separately as: $\int_0^3 (3x^2 + 2x + 1)\ dx = \int_0^3 3x^2\ dx + \int_0^3 2x\ dx + \int_0^3 1\ dx$ $= [x^3]_0^3 + [x^2]_0^3 + [x]_0^3$	1
	Determines the final value by substituting the limits as: $\text{Total Bicycles} = [x^3]_0^3 + [x^2]_0^3 + [x]_0^3 = (27-0) + (9-0) + (3-0) = 39 \quad \text{bicycles}$	0.5
29	**Evaluation Criteria:** Sets up the integral as: $\text{Average Value} = \frac{1}{6}\int_0^6 \left(1 + 2x - \frac{x^2}{3}\right) dx$	0.5
	Evaluates the integral as: $\int \left(1 + 2x - \frac{x^2}{3}\right) dx = \int 1\ dx + \int 2x\ dx - \int \frac{x^2}{3} dx$ $\int \left(1 + 2x - \frac{x^2}{3}\right) dx = x + x^2 - \frac{x^3}{9}$	1
	Applies the limits of integration as:	1

	$\left[x + x^2 - \frac{x^3}{3}\right]_0^6 = \left(6 + 36 - \frac{216}{9}\right) = 42 - 24 = 18$	
	Calculates the average value as: $\text{Average Value} = \frac{1}{6} \cdot 18 = 3\ ppm$ Concludes that the average pollution concentration in the river between 0 and 6 kilometres is 3 ppm.	0.5

Differential equations

30	A differential equation is given below where $k > 0$ is a constant: $\frac{dC}{dt} = -kC^2$ (i) Determine the order and degree of the differential equation. (ii) Explain why this differential equation does not represent a linear differential equation. [Skill: Understanding]	2
31	A leaky tank initially contains 10 litres of water. The rate at which water leaks out of the tank is proportional to the amount of water present at any time t. The differential equation governing the system is $\frac{dV}{dt} = -kV$, where V is the volume of water (in litres), $k > 0$ is a constant, and t is measured in minutes. (i) Solve the differential equation to find the volume of water $V(t)$ at any time t. (ii) If $k = 0.1$, find the volume of water in the tank after 5 minutes. *(Note: Use $e^{-0.5} \approx 0.61$.)* [Skill: Application]	3
	Answer the following 3 questions based on the information provided below. A social media influencer observes how their posts reach followers over time. The rate at which the number of people $P(t)$ who see the post increases is proportional to the difference between a fixed maximum potential audience size of 23 and the current number of people who have seen it. This phenomenon can be modelled by the following differential equation: $\frac{dP}{dt} = k(23 - P)$ where $P(t)$ is the number of people (in thousands) who have seen the post at time t (in hours), and k is a proportionality constant that depends on the rate of sharing. Initially, $P(0) = 0$.	
32	State the order and degree of the given differential equation with explanation. [Skill: Mechanical]	1
33	The influencer is seeking out an efficient method to solve the given differential equation. What is the most efficient method for solving it and what is the first step of that method? [Skill: Understanding]	1
34	Derive the function $P(t)$, which models the number of people (in thousands) who have seen the influencer's post at time t, in terms of t and k. [Skill: Application]	2

Marking Scheme

Q No.	Rubric	Marks
30	**Evaluation Criteria:** (i) Writes that the order of a differential equation is 1 and that the degree of a differential equation is also 1.	1
	(ii) Argues that the differential equation is not linear because it cannot be expressed in the standard form: $\frac{dC}{dt} + f(t)C = g(t)$ where *f(t)* and *g(t)* are either constants or functions of *t* only. The term -kC^2 introduces non-linearity because it involves *C* raised to a power greater than 1. Linear differential equations only involve terms that are linear in the dependent variable and its derivatives. OR Argues that the differential equation is not linear because it contains the dependent variable *C* is raised to a power greater than 1. Thus, this equation represents a first-order non-linear differential equation.	1
31	**Evaluation Criteria:** (i) Solves the differential equation by rearranging the equation and integrating both sides as: $\frac{dV}{V} = -kdt$ $\Rightarrow \int \frac{dV}{V} = -\int k\ dt$ $\Rightarrow \ln\lvert V\rvert = -kt + C$ $\Rightarrow V = e^C e^{-kt}$ (Award full marks if log is used instead of ln.)	1
	Uses the given initial conditions as: At *t* = 0, *V*(0) = 10 L, therefore $V(0) = e^C e^{-k\cdot 0} = 10$ $\Rightarrow e^C = 10$ Concludes the solution of the differential equation as: $V(t) = 10e^{-kt}$	1
	(ii) Substitutes *k* = 0.1 in the solution of step 2. $V(t) = 10e^{-0.1t}$	1

	At $t = 5$ mins, $V(5) = 10e^{-0.5} \approx 10 \times 0.61 \approx 6.1L$	
32	**Evaluation Criteria:** States that the highest derivative in the equation is $\frac{dP}{dt}$, so the order is 1.	0.5
	States that since the degree of the differential equation is the power of the highest-order derivative when it is free from radicals or fractions. Here, $\frac{dP}{dt}$ has an exponent/power of 1. Concludes that the degree is 1.	0.5
33	**Evaluation Criteria:** States that for the differential equation: $\frac{dP}{dt} = k(23 - P),$ the most efficient method for solving this is separation of variables.	0.5
	Argues that since this is a first-order linear differential equation, it can be rearranged so that all terms involving *P* are on one side, and all terms involving *t* are on the other side. $\frac{dP}{23 - P} = k\,dt$ Now, the equation is in a form where variables have been separated and it can now be integrated on both sides to find the solution.	0.5
34	**Evaluation Criteria:** Separates the variables as: $\frac{1}{23 - P}\,dP = k\,dt$ Integrates both sides: $\int \frac{1}{23 - P}\,dP = \int k\ dt$ $-\log\lvert 23 - P\rvert = kt + C$	0.5
	Exponentiates both sides to eliminate the logarithm: $23 - P = e^{-kt+C}$ $P = 23 - e^{-kt+C}$ $P = 23 - Me^{-kt}$ $where, M = e^{C}$	1
	Applies the initial condition $P(0) = 0$: $0 = 23 - Me^{-k\times 0} = 23 - M \Rightarrow M = 23$	0.5

	Writes the final function that models the number of people who have seen the influencer's post at time t as: $P(t) = 23 - 23e^{-kt} \Rightarrow P(t) = 23(1 - e^{-kt})$	

Vector algebra

35	Which of the following represents a unit vector in the direction of the vector given below? $3\hat{i} + 4\hat{j}$ (A) $\hat{i} + \hat{j}$ (B) $\frac{1}{5}\hat{i} + \frac{1}{5}\hat{j}$ (C) $\frac{3}{5}\hat{i} + \frac{4}{5}\hat{j}$ (D) $\frac{3}{5}\hat{j} + \frac{4}{5}\hat{i}$ [Skill: Mechanical]	1
36	A construction company is designing a new cable system to connect three points (A, B, and C) for a hanging bridge. The points are located in three-dimensional space, and the company needs to ensure that the cable between the three points is in a straight line for the system to be structurally stable. The coordinates of the points are: Point A: (1, 2, 7) Point B: (2, 6, 3) Point C: (3, 10, -1) Verify if the cable between the three points A, B, and C are in a straight line. [Skill: Understanding]	2
37	A company is designing a conveyor belt that will rotate objects. The force exerted by the belt is represented by $\overrightarrow{F} = 3\hat{i} + 2\hat{j} - \hat{k}$, and the displacement of the object on the belt is given by $\overrightarrow{d} = 2\hat{i} - \hat{j} - 4\hat{k}$ Find the projection of the force $\overrightarrow{F}$ vector along the displacement vector $\overrightarrow{d}$. [Skill: Application]	2
38	Two points are located at A(1, 2, 3) and B(4, 6, 8), with another point R(3, 3, 3) between them. (i) Verify whether the angle between the segments $\overrightarrow{AR}$ and $\overrightarrow{RB}$ is 180°. (ii) Compute the vector $\overrightarrow{AR} \times \overrightarrow{RB}$ and determine the angle between this resultant vector with both $\overrightarrow{AR}$ and $\overrightarrow{RB}$. [Skill: Understanding]	

Marking Scheme

Q No.	Rubric	Marks
35	**Correct Answer:** C The unit vector in the direction of the given vector is a vector divided by the magnitude of the given vector. $\frac{3\hat{i}+4\hat{j}}{\sqrt{3^2+4^2}} = \frac{3\hat{i}+4\hat{j}}{\sqrt{25}} = \frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}$	1
	A: Students selecting this option might assume that all unit vectors must have values of 1 for each coordinate.	
	B: Students selecting this option might have assumed that unit vector has coordinate values of 1 divided by the magnitude of the given vector.	
	D: Students selecting this might have not paid attention to the i and j components of the answer.	
36	**Evaluation Criteria:** Argues that to show that the points A, B, and C are in straight line (or collinear), it is sufficient to check if the vectors $\overrightarrow{AB}$ and $\overrightarrow{AC}$ are parallel Finds the vector $\overrightarrow{AB}$: $\overrightarrow{AB} = B - A = (2-1, 6-2, 3-7) = (1,4,-4)$ Finds the vector $\overrightarrow{AC}$: $\overrightarrow{AC} = C - A = (3-1, 10-2, -1-7) = (2,8,-8)$	0.5
	Finds the angle between the two vectors: $\cos\theta = \frac{\overrightarrow{AB}\cdot\overrightarrow{AC}}{\lvert\overrightarrow{AB}\rvert\lvert\overrightarrow{AC}\rvert}$ $\cos\theta = \frac{1\times 2 + 4\times 8 + (-4)\times(-8)}{\lvert\sqrt{1^2+4^2+4^2}\rvert\lvert\sqrt{2^2+8^2+8^2}\rvert}$ $\cos\theta = \frac{2+32+32}{\lvert\sqrt{33}\rvert\lvert\sqrt{132}\rvert}$ $\cos\theta = \frac{66}{\lvert\sqrt{33}\rvert\lvert\sqrt{132}\rvert}$ $\cos\theta = \frac{66}{66} = 1$ Argues that since $\cos(\theta) = 1$, the angle between the two vectors is 0° and hence the three points are collinear or are in straight line.	1.5

	Alternate solutions (award full marks if the student uses these methods): • Shows that $\overrightarrow{AB} = \overrightarrow{BC}$ • Shows that $\overrightarrow{AB} = \frac{1}{2}\overrightarrow{AC}$	
37	**Evaluation Criteria:** Writes that the projection of $\vec{F}$ along $\vec{d}$ is given by: $Proj_d(\vec{F}) = \frac{\vec{F} \cdot \vec{d}}{\vec{d}}$ Finds the dot product as: $\vec{F} \cdot \vec{d} = (3)(2) + (2)(-1) + (-1)(-4) = 6 - 2 + 4 = 8$	1
	Finds the magnitude of $\vec{d}$: $\lvert\vec{d}\rvert = \sqrt{(2)^2 + (-1)^2 + (-4)^2} = \sqrt{4 + 1 + 16} = \sqrt{21}$	0.5
	Computes the projection: $Proj_d(\vec{F}) = \frac{8}{\sqrt{21}}$	0.5
38	**Evaluation Criteria:** (i) Finds the vectors of $\overrightarrow{AR}$ and $\overrightarrow{RB}$ as: $\overrightarrow{AR} = \vec{R} - \vec{A} = (3-1)\hat{\imath} + (3-2)\hat{\jmath} + (3-3)\hat{k} = 2\hat{\imath} + \hat{\jmath}$ $\overrightarrow{RB} = \vec{B} - \vec{R} = (4-3)\hat{\imath} + (6-3)\hat{\jmath} + (8-3)\hat{k} = \hat{\imath} + 3\hat{\jmath} + 5\hat{k}$	1
	Finds their magnitudes as: $\lvert\overrightarrow{AR}\rvert = \sqrt{(2)^2 + (1)^2 + (0)^2} = \sqrt{4+1} = \sqrt{5}$ $\lvert\overrightarrow{RB}\rvert = \sqrt{(1)^2 + (3)^2 + (5)^2} = \sqrt{1+9+25} = \sqrt{35}$	1
	Finds the cosine of the angle as: $\cos\theta = \frac{\overrightarrow{AR} \cdot \overrightarrow{RB}}{\lvert\overrightarrow{AR}\rvert\lvert\overrightarrow{RB}\rvert}$ $\cos\theta = \frac{5}{\sqrt{5}\cdot\sqrt{35}} = \frac{5}{\sqrt{175}} = \frac{5}{5\sqrt{7}} = \frac{1}{\sqrt{7}}$ Concludes that since $\cos^{-1}\left(\frac{1}{\sqrt{7}}\right) \neq 180°$, the angle between the segments $\overrightarrow{AR}$ *and* $\overrightarrow{RB}$is not 180°.	0.5
	(ii) Finds the cross product of two vectors as: $\overrightarrow{AR} \times \overrightarrow{RB} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 2 & 1 & 0 \\ 1 & 3 & 5 \end{vmatrix}$	2

	$= 5\hat{\imath} - 10\hat{\jmath} + 5\hat{k}$	
	Writes that the since the cross-product vector is perpendicular to the plane comprising of the original vectors, this vector is perpendicular to both $\overrightarrow{AR}$ and $\overrightarrow{RB}$.	0.5

Three-dimensional geometry

39	Given below is a straight line in its vector form where μ is a parameter. $r = \left(2\hat{i} - \hat{j} + 4\hat{k}\right) + \mu\left(3\hat{i} - 4\hat{j} + \hat{k}\right)$ Which of the following is a point on this line? (i) (2, -1, 4) (ii) (3, -4, 1) (iii) (5, -5, 5) (A) only (i) (B) only (i) and (ii) (C) only (i) and (iii) (D) all - (i), (ii), and (iii) [Skill: Application]	1
40	Two lines are given below where *m* is a constant: $L_1 : \dfrac{x-2}{1} = \dfrac{y-3}{m} = \dfrac{z-4}{m+1}$ $L_2 : \dfrac{x-1}{-1} = \dfrac{y-4}{m+1} = \dfrac{z-5}{m}$ Find the value of *m* such that the two lines are at an angle of 60° to each other. [Skill: Understanding]	3
41	Two airplanes are flying on parallel paths in 3D space. The first airplane's flight path is given by the equation: $\dfrac{x+1}{4} = \dfrac{y-3}{5} = \dfrac{z+2}{2}$ The second airplane passes through the point $2\hat{i} + 6\hat{j} + \hat{k}$ and follows a flight path parallel to the first one. Write the equation of the second airplane, parallel to the first airplane, and find the shortest distance between the two airplanes. [Skill: Application]	3

Marking Scheme

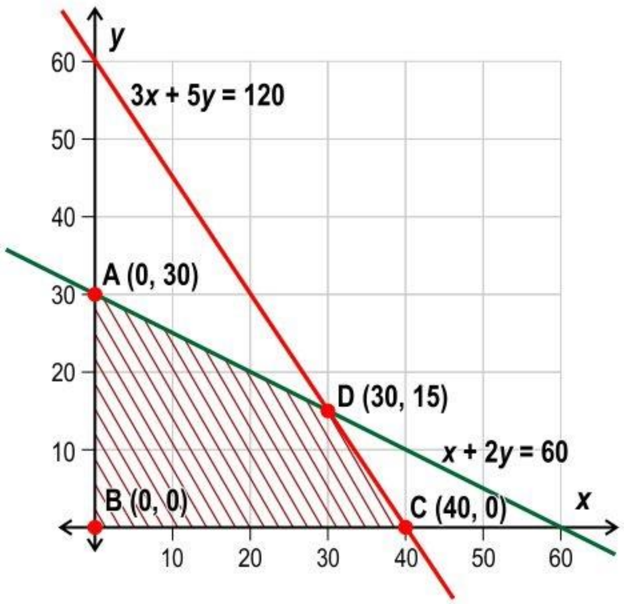
Q No.	Rubric	Marks
39	**Correct Answer:** C Any point shall be the line if there exists an integral value of μ for which the value of r equals to the point. For (i) - μ is 0 and for (iii), μ is 1. There doesn't exist any value of μ for the (ii).	1
	A: Students selecting this option might have incorrectly neglected the (iii) point by assuming that only the point in the line definition lies on the line.	
	B: Students selecting this option might have incorrectly assumed that both vectors in the definition of the line (separated by μ) represent points in 3D space that lie of the given line.	
	D: Students selecting this option might have made the cumulative errors from both option A and option B.	
40	**Evaluation Criteria:** States that the dot product of the direction ratios of two lines is equal to $cos(60°)$. $$\frac{\overrightarrow{b_1} \cdot \overrightarrow{b_2}}{\lvert\overrightarrow{b_1}\rvert \cdot \lvert\overrightarrow{b_2}\rvert} = \cos(60°) = \frac{1}{2}$$ where $\overrightarrow{b_1} = \hat{\imath} + m\hat{\jmath} + (m+1)\hat{k}$ $\overrightarrow{b_2} = -\hat{\imath} + (m+1)\hat{\jmath} + m\hat{k}$	0.5
	Determines the dot product in term of m: $\overrightarrow{b_1} \cdot \overrightarrow{b_2} = -1 + m(m+1) + (m+1)m = -1 + m^2 + m + m^2 + m = 2m^2 + 2m - 1$	1
	Determines the magnitudes of the two vectors: $\lvert\overrightarrow{b_1}\rvert = \sqrt{1 + m^2 + (m+1)^2}$ $\lvert\overrightarrow{b_1}\rvert = \sqrt{2m^2 + 2m + 2}$ $\lvert\overrightarrow{b_2}\rvert = \sqrt{1 + (m+1)^2 + m^2}$ $\lvert\overrightarrow{b_2}\rvert = \sqrt{2m^2 + 2m + 2}$	0.5
	Finds m by solving the equation: $$\frac{2m^2 + 2m - 1}{\sqrt{2m^2 + 2m + 2} \cdot \sqrt{2m^2 + 2m + 2}} = \frac{1}{2}$$	1

	$\Rightarrow \frac{2m^2+2m-1}{2m^2+2m+2} = \frac{1}{2}$ $\Rightarrow m^2+m-2=0$ Concludes that the values of *m* are 1 and (-2).	
41	**Evaluation Criteria:** Argues that since the second airplane passes through (2,6,1) and travels in the parallel direction to the first one with direction ratios (4, 5, 2), hence the equation for the second plane is: $\frac{x-2}{4} = \frac{y-6}{5} = \frac{z-1}{2}$	0.5
	Writes the equation for shortest distance as: $d = \frac{\lvert \vec{b} \times (a_2 - a_1) \rvert}{\lvert \vec{b} \rvert}$ where $\vec{b} = 4\hat{\imath} + 5\hat{\jmath} + 2\hat{k}$ $\overrightarrow{a_2} = 2\hat{\imath} + 6\hat{\jmath} + \hat{k}$ $\overrightarrow{a_1} = -\hat{\imath} + 3\hat{\jmath} - 2\hat{k}$ $\vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1}) = 3(3\hat{\imath} - 2\hat{\jmath} - \hat{k})$	1
	Computes the magnitude of the cross product as: $\lvert \vec{b} \times (\overrightarrow{a_2} - \overrightarrow{a_1}) \rvert = 3\sqrt{3^2+(-2)^2+(-1)^2} = 3\sqrt{9+4+1} = 3\sqrt{14}$	1
	Computes the shortest distance as: $d = \frac{3\sqrt{14}}{\sqrt{4^2+5^2+2^2}} = \frac{3\sqrt{14}}{\sqrt{45}} = \frac{3\sqrt{14}}{3\sqrt{5}} = \frac{\sqrt{14}}{\sqrt{5}} = \sqrt{\frac{14}{5}}$units	0.5

Linear programming problem

42	Which of the following statements is correct with reference to a linear programming problem? (A) A linear programming problem whose feasible region is unbounded cannot have an optimal solution. (B) It is possible for a non-corner point to have the optimal value of an objective function. (C) A linear programming problem can have only finitely many optimal solutions. (D) The feasible region in a linear programming problem cannot be empty. [Skill: Application]	1
43	Solve the linear programming problem graphically: Maximise: $Z = 3x + 5y$ subject to the constraints: $3x + 2y \leq 120$ $x + 2y \leq 60$ $x \geq 0$ $y \geq 0$ [Skill: Mechanical]	5

Marking Scheme

<table>
<tr><td>Q No.</td><td>Rubric</td><td>Marks</td></tr>
<tr><td>42</td><td>Correct Answer: B
If two adjacent corner points yield the optimal value for the objective function, then every point on the line joining them also yields the optimal value. Hence, it is possible for a non-corner point to have the optimal value of an objective function.</td><td>1</td></tr>
<tr><td></td><td>A: Students selecting this option may think that an unbounded feasible region always results in no optimal solution.</td><td></td></tr>
<tr><td></td><td>C: Students selecting this option may not know that an LPP can have infinitely many solutions.</td><td></td></tr>
<tr><td></td><td>D: Students selecting this option may think that feasible region is always non-empty.</td><td></td></tr>
<tr><td>43</td><td>Evaluation Criteria:
Draws the graph of the system of inequalities and shades the feasible region. The graph may look as follows:


(Award 1 mark each for drawing the lines and 0.5 marks for locating the corner points.)</td><td>2.5</td></tr>
<tr><td></td><td>Evaluates the objective function at the corner points as follows:

Corner points (x, y)	$Z = 3x + 5y$
A (0, 30)	150
B (0,0)	0
C (40, 0)	120
D (30, 15)	165

</td><td>2</td></tr>
<tr><td></td><td>Identifies the maximum value of Z as 165. Hence, concludes that the maximum value of the objective function occurs at the extreme point (30, 15) and the optimal solution to the given LP problem is: $x = 30$, $y = 15$ and max Z = 165.</td><td>0.5</td></tr>
</table>

Probability

44	In a town, 30% of the residents have a cat, 40% have a dog, and 15% have both a cat and a dog. A random resident is selected, and it is found that they own a dog. Given that the person has a dog, what is the probability that they also have a cat? (A) $\frac{15}{40}$ (B) $\frac{15}{30}$ (C) $\frac{30}{40}$ (D) $\frac{30}{70}$ [Skill: Application]	1
45	A card is randomly drawn from a standard deck of 52 cards. Two events are defined as follows: A: The card is a heart. B: The card is a face card (Jack, Queen, King). Determine whether A and B are independent events. Justify your answer. [Skill: Understanding]	2
46	A factory produces three types of products A, B, and C. The probabilities of randomly selecting a product from a batch of products are P(A) = 0.4, P(B) = 0.35, and P(C) = 0.25. A defect is found in 2% of product A, 3% of product B, and 4% of product C. If a randomly selected product is found to be defective, what is the probability that it is product B? [Skill: Application]	3
47	At a college, students are required to choose one of three subjects for their elective course: Mathematics, Physics, or Biology. The probabilities of a student selecting each subject are: $P(Math) = 0.3, P(Physics) = 0.5, P(Biology) = 0.2$ It is observed that the likelihood of a student failing the elective course depends on the subject chosen. The probabilities of a student failing given their choice of subject are: $P(Fail\|Math) = 0.02, P(Fail\|Physics) = 0.03, P(Fail\|Biology) = 0.05$ Find the probability that a randomly selected student fails their elective course. [Skill: Mechanical]	3

	Answer the following 3 questions based on the given information. A traffic monitoring agency is analyzing the road accidents on bad weather days. They collected the following data: 1. 70% of accidents involve cars, while the remaining 30% involve motorcycles. 2. The probability that the weather was bad given that there was a car accident is 0.6. That is, $P(BadWeather\|Car) = 0.6$. 3. The probability that the weather was bad given that there was a motorcycle accident is 0.8. That is, $P(BadWeather\|Motorcycle) = 0.8$.	
48	A day of accidents is selected at random. What is the probability that the weather on that day is bad? [Skill: Application]	2
49	Determine whether the events "Bad Weather" and "Car Accident" are independent, using appropriate probabilities. [Skill: Understanding]	1
50	What is the probability that an accident involves a car, given that it occurred during bad weather? [Skill: Understanding]	1

Marking Scheme

QNo.	Rubric	Marks
44	**Correct Answer:** A $P(\text{Cat} \mid \text{Dog}) = \frac{P(\text{Cat} \cap \text{Dog})}{P(\text{Dog})}$ $\implies P(\text{Cat} \mid \text{Dog}) = \frac{0.15}{0.40} = \frac{15}{40}$	1
	B: Students selecting this might have incorrectly used the following expression for conditional probability $P(\text{Cat} \mid \text{Dog}) = \frac{P(\text{Cat} \cap \text{Dog})}{P(\text{Cat})}$	
	C: Students selecting this might have incorrectly assumed this to be the expression for conditional probability - $P(\text{Cat} \mid \text{Dog}) = \frac{P(\text{Cat})}{P(\text{Dog})}$	
	D: Students selecting this might have incorrectly assumed this to be the expression for conditional probability - $P(\text{Cat} \mid \text{Dog}) = \frac{P(\text{Cat})}{P(\text{Cat}) + P(\text{Dog})}$	
45	**Evaluation Criteria:** Identifies the number of cards that are relevant to the events as: Total cards in the deck = 52 Total hearts = 13 Total face cards = 12 Calculates P(A), P(B) and $P(A \cap B)$ as: $P(A) = \frac{\text{Number of hearts}}{\text{Total cards}} = \frac{13}{52} = \frac{1}{4}.$ $P(B) = \frac{\text{Number of face cards}}{\text{Total cards}} = \frac{12}{52} = \frac{3}{13}.$ $P(A \cap B) = \frac{\text{Number of face cards that are hearts}}{\text{Total cards}} = \frac{3}{52}.$	1
	Checks for independence:	1

	$P(A)\cdot P(B)=\frac{1}{4}\cdot\frac{3}{13}=\frac{3}{52}=P(A\cap B)$ Since $P(A\cap B)=P(A)\cdot P(B)$, concludes that A and B are independent events.	
46	**Evaluation Criteria:** Uses Bayes' Theorem: $P(B\|\text{Defective})=\frac{P(B)\cdot P(\text{Defective}\|B)}{P(\text{Defective})}$ Notes that $P(\text{Defective}\|B)=0.03$	0.5
	Uses the total probability theorem to write: $P(\text{Defective})=P(A)\cdot P(\text{Defective}\|A)+P(B)\cdot P(\text{Defective}\|B)+P(C)\cdot P(\text{Defective}\|C)$ $\Rightarrow\left(\frac{40}{100}\cdot\frac{2}{100}\right)+\left(\frac{35}{100}\cdot\frac{3}{100}\right)+\left(\frac{25}{100}\cdot\frac{4}{100}\right)$ $\Rightarrow\frac{80}{10000}+\frac{105}{10000}+\frac{100}{10000}=\frac{285}{10000}$ or 0.0285	1.5
	Calculates $P(B\|\text{Defective})$: $P(B\|\text{Defective})=\frac{\left(\frac{35}{100}\right)\left(\frac{3}{100}\right)}{\frac{285}{10000}}=\frac{105}{285}=\frac{7}{19}$	1
47	**Evaluation Criteria:** Recalls the theorem of total probability $P(\text{Fail})=P(\text{ Fail } \mid \text{ Math })P(\text{Math})+P(\text{ Fail } \mid \text{ Physics })P(\text{Physics})+P(\text{ Fail } \mid \text{ Biology })P(\text{Biology})$	1
	Calculates the probability: $P(\text{Fail})=(0.02)(0.3)+(0.03)(0.5)+(0.05)(0.2)$ P(D) = 0.006 + 0.015 + 0.01 = 0.031 Concludes that randomly selected student fails their elective course is 0.031 or 3.1%	2
48	**Evaluation Criteria:** Uses conditional probability to find: $P(\text{Motorcycle}\cap\text{Bad Weather})=P(\text{Motorcycle})\cdot P(\text{Bad Weather}\|\text{Motorcycle})$ $\Rightarrow P(\text{Motorcycle}\cap\text{Bad Weather})=0.3\cdot 0.8=0.24$ $P(\text{Car}\cap\text{Bad Weather})=P(\text{Car})\cdot P(\text{Bad Weather}\|\text{Car})$	1

	$\Rightarrow P(\text{Car} \cap \text{Bad Weather}) = 0.7 \cdot 0.6 = 0.42$	
	Calculates the probability to find $P(\text{Bad Weather})$ as: $P(\text{Bad Weather}) = P(\text{Car} \cap \text{Bad Weather}) + P(\text{Motorcycle} \cap \text{Bad Weather})$ $\Rightarrow P(\text{Bad Weather}) = 0.42 + 0.24 = 0.66$	1
49	**Evaluation Criteria:** Tests independence condition: $P(\text{Bad Weather}) \cdot P(\text{Car}) = 0.66 \cdot 0.7 = 0.462.$ $P(\text{Car} \cap \text{Bad Weather}) = 0.42$ Argues that since $P(\text{Bad Weather} \cap \text{Car}) = 0.42 \neq 0.462$, the events are not independent.	1
50	**Evaluation Criteria:** Recalls the relevant probabilities from previous questions: $P(\text{Bad Weather}) = 0.66.$ $P(\text{Car} \cap \text{Bad Weather}) = 0.42.$ $Calculates\ the\ required\ probability\ as$: $P(\text{Car}\|\text{Bad Weather}) = \frac{P(\text{Car} \cap \text{Bad Weather})}{P(\text{Bad Weather})}$ $\Rightarrow P(\text{Car}\|\text{Bad Weather}) = \frac{0.42}{0.66} = \frac{7}{11} \approx 0.636$	1

Board of School Education Haryana, Bhiwani