

CLASS : 12th (Sr. Secondary)

4381/4331

Series : SS-M/2019

Total No. of Printed Pages : 64

SET : A, B, C & D

MARKING INSTRUCTIONS AND MODEL ANSWERS

MATHEMATICS

ACADEMIC/OPEN

(Only for Fresh/Re-appear Candidates)

उप-परीक्षक मूल्यांकन निर्देशों का ध्यानपूर्वक अवलोकन करके उत्तर-पुस्तिकाओं का मूल्यांकन करें। यदि परीक्षार्थी ने प्रश्न पूर्ण व सही हल किया है तो उसके पूर्ण अंक दें।

General Instructions :

- (i) *Examiners are advised to go through the general as well as specific instructions before taking up evaluation of the answer-books.*
- (ii) *Instructions given in the marking scheme are to be followed strictly so that there may be uniformity in evaluation.*
- (iii) *Mistakes in the answers are to be underlined or encircled.*
- (iv) *Examiners need not hesitate in awarding full marks to the examinee if the answer/s is/are absolutely correct.*
- (v) *Examiners are requested to ensure that every answer is seriously and honestly gone through before it is awarded mark/s. It will ensure the authenticity as their evaluation and enhance the reputation of the Institution.*

4381/4331/(Set : A, B, C & D)

P. T. O.

- (vi) A question having parts is to be evaluated and awarded partwise.
- (vii) If an examinee writes an acceptable answer which is not given in the marking scheme, he or she may be awarded marks only after consultation with the head-examiner.
- (viii) If an examinee attempts an extra question, that answer deserving higher award should be retained and the other scored out.
- (ix) Word limit wherever prescribed, if violated up to 10%. On both sides, may be ignored. If the violation exceeds 10%, 1 mark may be deducted.
- (x) Head-examiners will approve the standard of marking of the examiners under them only after ensuring the non-violation of the instructions given in the marking scheme.
- (xi) Head-examiners and examiners are once again requested and advised to ensure the authenticity of their evaluation by going through the answers seriously, sincerely and honestly. The advice, if not heeded to, will bring a bad name to them and the Institution.

महत्त्वपूर्ण निर्देश :

- (i) अंक-योजना का उद्देश्य मूल्यांकन को अधिकाधिक वस्तुनिष्ठ बनाना है। अंक-योजना में दिए गए उत्तर-बिन्दु अंतिम नहीं हैं। ये सुझावात्मक एवं सांकेतिक हैं। यदि परीक्षार्थी ने इनसे भिन्न, किन्तु उपयुक्त उत्तर दिए हैं, तो उसे उपयुक्त अंक दिए जाएँ।

- (ii) शुद्ध, सार्थक एवं सटीक उत्तरों को यथायोग्य अधिमान दिए जाएँ।
- (iii) परीक्षार्थी द्वारा अपेक्षा के अनुरूप सही उत्तर लिखने पर उसे पूर्णांक दिए जाएँ।
- (iv) वर्तनीगत अशुद्धियों एवं विषयांतर की स्थिति में अधिक अंक देकर प्रोत्साहित न करें।
- (v) भाषा-क्षमता एवं अभिव्यक्ति-कौशल पर ध्यान दिया जाए।
- (vi) मुख्य-परीक्षकों/उप-परीक्षकों को उत्तर-पुस्तिकाओं का मूल्यांकन करने के लिए केवल Marking Instructions/ Guidelines दी जा रही है यदि मूल्यांकन निर्देश में किसी प्रकार की त्रुटि हो, प्रश्न का उत्तर स्पष्ट न हो, मूल्यांकन निर्देश में दिए गए उत्तर से अलग कोई और भी उत्तर सही हो तो परीक्षक, मुख्य-परीक्षक से विचार-विमर्श करके उस प्रश्न का मूल्यांकन अपने विवेक अनुसार करें।

SET - A**SECTION - A**

1. (i) $f \circ g(x) = f(3x^2)$
 $= f(3x^2)$
 $= \cos(3x^2)$ 1
- (ii) $\sin^{-1}\left(-\frac{1}{2}\right) = \sin^{-1}\left(-\sin\frac{\pi}{6}\right)$
 $= \sin^{-1}\sin\left(-\frac{\pi}{6}\right) = -\frac{\pi}{6}$

Ans. (B) 1

(4)

4381/4331

$$(iii) \quad a_{ij} = \frac{1}{2} |i - 3j|$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix} = \begin{bmatrix} 1 & \frac{5}{2} \\ \frac{1}{2} & 2 \\ 0 & \frac{3}{2} \end{bmatrix} \quad 1$$

$$(iv) \quad \begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

$$= 3 - x^2 = 3 - 8$$

$$= x^2 = 8 \Rightarrow x = \pm 2\sqrt{2} \quad \text{Ans. (C)} \quad 1$$

$$(v) \quad \frac{d}{dx} \sin(x^2 + 5) = \cos(x^2 + 5) \frac{d}{dx} (x^2 + 5) \\ = 2x \cos(x^2 + 5) \quad 1$$

$$(vi) \quad \frac{d}{dr} (A) = \frac{d}{dr} (\pi r^2) \\ = 2\pi r$$

$$\text{at } r = 3, \quad \frac{dA}{dr} = 6\pi \text{ cm}^2 / \text{sec} \quad \text{Ans. (A)} \quad 1$$

$$(vii) \quad y = x^3 - x$$

$$\frac{dy}{dx} = 3x^2 - 1$$

4381/4331/(Set : A, B, C & D)

(5)

4381/4331

$$\text{at } x = 2, \frac{dy}{dx} = 3 \times 4 - 1 = 11$$

$$\therefore \text{ slope of tangent at } x = 2 = 11 \quad 1$$

(viii) put $\tan^{-1} x = t$

$$\frac{1}{1+x^2} dx = dt$$

$$\begin{aligned} \therefore \int \frac{\sin(\tan^{-1} x)}{1+x^2} dx &= \int \sin t \, dt \\ &= -\cos t + c \\ &= -\cos(\tan^{-1} x) + c \quad 1 \end{aligned}$$

(ix) $f(x) = \sin^5 x \cos^4 x$

$$\begin{aligned} \therefore f(-x) &= \{\sin(-x)\}^5 \{\cos(-x)\}^4 \\ &= -\sin^5 x \cos^4 x = -f(x) \end{aligned}$$

$\therefore f_1$ is odd function

$$\therefore \int_{-1}^1 \sin^5 x \cos^4 x \, dx = 0 \quad \text{Ans. (C) } 1$$

(x) Order = 2 **Ans. (A) 1**

(xi) degree = 1 **Ans. (C) 1**

4381/4331/(Set : A, B, C & D)

P. T. O.

(6)

4381/4331

$$\begin{aligned} \text{(xii) } P(E/F) &= \frac{P(E \cap F)}{P(F)} \\ &= \frac{0.2}{0.3} = \frac{2}{3} \end{aligned} \quad 1$$

$$\begin{aligned} \text{(xii) } P(A \text{ and } B) &= P(A \cap B) \\ &= P(A) \cdot P(B) \\ &= 0.3 \times 0.6 = 0.18 \end{aligned} \quad 1$$

(xiv) 4 white balls, 6 black balls.

$$P(\text{both black}) = \frac{6}{10} \times \frac{6}{10} = \frac{9}{25} \quad 1$$

$$\begin{aligned} \text{(xv) } \vec{a} + \vec{b} &= (2\hat{i} + 2\hat{j} - 5\hat{k}) + (2\hat{i} + \hat{j} + 3\hat{k}) \\ &= 4\hat{i} + 3\hat{j} - 2\hat{k} \\ |\vec{a} + \vec{b}| &= \sqrt{16 + 9 + 4} = \sqrt{29} \\ \text{Unit vector of } \vec{a} + \vec{b} &= \frac{1}{\sqrt{29}} [4\hat{i} + 3\hat{j} - 2\hat{k}] \end{aligned} \quad 1$$

(xvi) D. R's of a line are 2, -1, -2

$$\begin{aligned} \therefore \text{D.C's of the line are } &\frac{2}{\sqrt{4+1+4}}, \frac{-1}{3}, \frac{-2}{3} \\ &= \frac{2}{3}, \frac{-1}{3}, \frac{-2}{3} \end{aligned} \quad 1$$

SECTION - B

$$2. \quad 5 * 7 = \text{L.C.M. of 5 and 7} = 35 \quad 2$$

4381/4331/(Set : A, B, C & D)

(7)

4381/4331

3. $\sin^{-1}\left(\sin \frac{3\pi}{5}\right) = \sin^{-1} \sin\left(\pi - \frac{2\pi}{5}\right)$ 1

$$= \sin^{-1} \sin\left(\frac{2\pi}{5}\right)$$
$$= \frac{2\pi}{5}$$

Principal value of \sin^{-1} 1

$$\frac{-\pi}{2} \leq x \leq \frac{\pi}{2}$$

4. $2A + B = 2 \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$ 1

$$= \begin{bmatrix} 4 & 8 \\ 6 & 4 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$
$$= \begin{bmatrix} 5 & 11 \\ 4 & 9 \end{bmatrix}$$
 1

5. $|A| = \begin{vmatrix} 1 & 2 \\ 4 & 2 \end{vmatrix} = 2 - 8 = -6$ 1

$$2A = 2 \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$|2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix} = 8 - 32 = -24$$

$$= 4x(-6) = 4|A|$$
 1

6. $2x + 3y = \sin y$

diff. w.r.t. x, we get

$$2 + 3 \frac{dy}{dx} = \cos y \frac{dy}{dx}$$
 1

4381/4331/(Set : A, B, C & D)

P. T. O.

$$\Rightarrow 2 = \frac{dy}{dx}(\cos y - 3)$$

$$\therefore \frac{dy}{dx} = \frac{2}{\cos y - 3} \quad 1$$

7. $x = 2at^2$

$$\frac{dx}{dt} = 4at \quad \dots\dots\dots \text{(i)} \quad \frac{1}{2}$$

$$y = at^4$$

$$\frac{dy}{dt} = 4at^3 \quad \dots\dots\dots \text{(ii)} \quad \frac{1}{2}$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{4at^3}{4at} = t^2 \quad 1$$

$$\begin{aligned} \text{8. } \int x \cdot \sin x dx &= x \cdot \int \sin x dx - \int \left[\frac{d}{dx}(x) \int \sin x dx \right] dx \quad 1 \\ &= -x \cos x + \int \cos x dx + c \\ &= -x \cos x + \sin x + c \quad 1 \end{aligned}$$

$$\begin{aligned} \text{9. } \int_2^3 \frac{1}{x} dx &= [\log x]_2^3 \quad 1 \\ &= \log 3 - \log 2 = \log \frac{3}{2} \quad 1 \end{aligned}$$

10. $\frac{x}{a} + \frac{y}{b} = 1 \quad \dots\dots\dots \text{(i)}$

diff. w. r. t. x , we get

4381/4331/(Set : A, B, C & D)

(9)

4381/4331

$$\frac{1}{a} + \frac{1}{b} \frac{dy}{dx} = 0 \dots\dots (ii) \quad \frac{1}{2}$$

Again diff. w.r.t. x, we get

$$\frac{1}{b} \cdot \frac{d^2y}{dx^2} = 0 \dots\dots\dots (iii) \quad \frac{1}{2}$$

$$\text{or } \frac{d^2y}{dx^2} = 0. \quad 1$$

$$11. \quad n = 10, p = \frac{1}{2}, q = 1 - \frac{1}{2} = \frac{1}{2} \quad 1$$

$$p(X = 6) = {}^n C_6 \left(\frac{1}{2}\right)^{n-6} \cdot \left(\frac{1}{2}\right)^6 \quad 1$$

$$= {}^{10} C_6 \left(\frac{1}{2}\right)^4 \cdot \left(\frac{1}{2}\right)^6$$

$$= \frac{10}{6 \cdot 4} \left(\frac{1}{2}\right)^{10} = \frac{105}{512} \quad 1$$

SECTION - C

$$12. \quad \tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$$

$$= \tan^{-1} \frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta} \quad \text{put } x = \tan\theta \quad 1$$

(10)

4381/4331

$$\begin{aligned} &= \tan^{-1}\left(\frac{\sec \theta - 1}{\tan \theta}\right) \\ &= \tan^{-1}\left(\frac{\frac{1}{\cos \theta} - 1}{\frac{\sin \theta}{\cos \theta}}\right) \\ &= \tan^{-1} \frac{1 - \cos \theta}{\sin \theta} && 1 \\ &= \tan^{-1} \frac{2 \sin^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} && 1 \\ &= \tan^{-1}\left(\tan \frac{\theta}{2}\right) = \frac{1}{2} \theta = \frac{1}{2} \tan^{-1} x && 1 \end{aligned}$$

13. $f\left(\frac{\pi}{2}\right) = 3$ 1

$$\begin{aligned} \lim_{x \rightarrow \frac{\pi}{2}} f(x) &= \lim_{h \rightarrow 0} \frac{K \cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)} \\ &= \lim_{h \rightarrow 0} \frac{-K \sin h}{-2h} \\ &= \lim_{h \rightarrow 0} \frac{K}{2} \cdot \frac{\sin h}{h} = \frac{K}{2} && 1 \end{aligned}$$

4381/4331/(Set : A, B, C & D)

(11)

4381/4331

f is continuous at $x = \frac{\pi}{2}$

$$\therefore \lim_{x \rightarrow \frac{\pi}{2}} f(x) = f\left(\frac{\pi}{2}\right) \quad 1$$

$$\therefore \frac{K}{2} = 3 \Rightarrow K = 6 \quad 1$$

14. $f(x) = 2x^3 - 3x^2 - 36x + 7$

$$f'(x) = 6x^2 - 6x - 36$$

for strictly increasing, $f'(x) > 0$

$$\therefore 6(x^2 - x - 6) > 0$$

$$\Rightarrow (x - 3)(x + 2) > 0 \quad 1$$

Case I : $x - 3 > 0$ and $x + 2 > 0$

$$\Rightarrow x > 3 \text{ and } x > -2$$

$$\Rightarrow x > 3 \text{ or } (3, \infty) \quad 1$$

Case II : $x - 3 < 0$ and $x + 2 < 0$

$$\Rightarrow x < 3 \text{ and } x < -2$$

$$\Rightarrow x < -2 \text{ or } (-\infty, -2) \quad 1$$

$\therefore f(x)$ is strictly increasing

$$\text{in } (3, \infty) \text{ and } (-\infty, -2) \quad 1$$

4381/4331/(Set : A, B, C & D)

P. T. O.

15. Let $A = \{4a \cos\}$ 1

$B = \{\text{Remaining 48 cards}\}$

$X = 0, 1, 2$

$$P(X = 0) = \frac{48}{52} \times \frac{48}{52} = \frac{144}{169} \quad 1$$

$$\begin{aligned} P(X = 1) &= \frac{4}{52} \times \frac{48}{52} + \frac{48}{52} \times \frac{4}{52} \\ &= \frac{1}{13} \times \frac{12}{13} + \frac{12}{13} \times \frac{1}{13} = \frac{24}{169} \quad 1 \end{aligned}$$

$$P(X = 2) = \frac{4}{52} \times \frac{4}{52} = \frac{1}{169} \quad 1$$

16. $\vec{a} + \vec{b} = (\hat{i} + \hat{j} + \hat{k}) + (\hat{i} + 2\hat{j} + 3\hat{k})$

$$= 2\hat{i} + 3\hat{j} + 4\hat{k}$$

$\vec{a} - \vec{b} = (\hat{i} + \hat{j} + \hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k})$

$$= -\hat{j} - 2\hat{k}$$

$$(\vec{a} + \vec{b}) \times (\vec{a} - \vec{b}) = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 3 & 4 \\ 0 & -1 & -2 \end{vmatrix}$$

$$= -2\hat{i} + 4\hat{j} - 2\hat{k} (= \vec{C}) \quad 2$$

$$\therefore |\vec{C}| = \sqrt{4 + 16 + 4} = \sqrt{24} = 2\sqrt{6} \quad 1$$

\therefore Required vector

$$\hat{C} = -\frac{1}{\sqrt{6}}\hat{i} + \frac{2}{\sqrt{6}}\hat{j} - \frac{1}{\sqrt{6}}\hat{k} \quad 1$$

SECTION - D

17. $x - y + z = 4$ (i)

$2x + y - 3z = 0$ (ii)

$x + y + z = 2$ (iii)

$AX = B$

$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$ 1

$\therefore X = A^{-1} B$ (iv)

$|A| = 1(1 + 3) + 1(2 + 3) + 1(2 - 1)$ 1

$= 4 + 5 + 1 = 10 \neq 0.$

$Adj A = \begin{bmatrix} 4 & -5 & 1 \\ 2 & 0 & -2 \\ 2 & 5 & 3 \end{bmatrix}' = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ 1

$\therefore A^{-1} = \frac{1}{|A|} Adj A = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$ 1

(14)

4381/4331

$$X = A^{-1}B = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix} = \frac{1}{10} \begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix} \quad 1$$

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

Hence $x = 2, y = -1, z = 1$ 1

18. $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ (i)

$$y = \frac{b}{a} \sqrt{a^2 - x^2} \quad \text{..... (ii)} \quad 1$$

Required Area = $4 \int_0^a y \, dx$ 1

$$= 4 \frac{b}{a} \int_0^a \sqrt{a^2 - x^2} \, dx$$

$$= \frac{4b}{a} \left[\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \right]_0^a \quad 1$$

$$= \frac{4b}{a} \left[0 + \frac{a^2}{2} \sin^{-1}(1) - 0 \right] \quad 1$$

$$= \frac{4b}{a} \times \frac{a^2}{2} \times \frac{\pi}{2} \quad 1$$

$$= \pi ab \quad 1$$

4381/4331/(Set : A, B, C & D)

OR

$$\text{Let } I = \int_0^{\pi/2} \frac{\cos^5 x}{\sin^5 x + \cos^5 x} dx \dots\dots\dots (i)$$

$$\therefore I = \int_0^{\pi/2} \frac{\cos^5\left(\frac{\pi}{2} - x\right)}{\sin^5\left(\frac{\pi}{2} - x\right) + \cos^5\left(\frac{\pi}{2} - x\right)} dx \quad 1$$

$$\Rightarrow I = \int_0^{\pi/2} \frac{\sin^5 x}{\cos^5 x + \sin^5 x} dx \dots\dots\dots (ii) \quad 1$$

Adding (i) and (ii)

$$2I = \int_0^{\pi/2} \frac{\sin^5 x + \cos^5 x}{\sin^5 x + \cos^5 x} dx \quad 1$$

$$= \int_0^{\pi/2} 1 \cdot dx = \left[x \right]_0^{\pi/2} = \frac{\pi}{2} \quad 1$$

$$\therefore I = \frac{\pi}{4} \quad 2$$

$$19. \vec{r} = \hat{i} + \hat{j} + \lambda(2\hat{i} - \hat{j} + \hat{k}) \dots\dots\dots (i)$$

$$\vec{r} = 2\hat{i} + \hat{j} - \hat{k} + \mu(3\hat{i} - 5\hat{j} + 2\hat{k}) \dots\dots\dots (ii)$$

$$\vec{a}_1 = \hat{i} + \hat{j}, \vec{b}_1 = 2\hat{i} - \hat{j} + \hat{k} \quad 1$$

$$\vec{a}_2 = 2\hat{i} + \hat{j} - \hat{k}, \vec{b}_2 = 3\hat{i} - 5\hat{j} + 2\hat{k} \quad 1$$

$$\vec{a}_2 - \vec{a}_1 = \hat{i} + 0\hat{j} - \hat{k} \quad 1$$

(16)

4381/4331

$$\vec{b}_1 \times \vec{b}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & -5 & 2 \end{vmatrix} = 3\hat{i} - \hat{j} - 7\hat{k} \quad 1$$

$$|\vec{b}_1 \times \vec{b}_2| = \sqrt{9+1+49} = \sqrt{59} \quad 1$$

$$\text{S.D.} = \left| \frac{(\vec{b}_1 \times \vec{b}_2) \cdot (\vec{a}_2 - \vec{a}_1)}{|\vec{b}_1 \times \vec{b}_2|} \right| = \left| \frac{3-0+7}{\sqrt{59}} \right| = \frac{10}{\sqrt{59}} \quad 1$$

OR

Equation of plane passing through the point (1,1, -1) is :

$$a(x - 1) + b(y - 1) + c(z + 1) = 0 \quad \dots\dots\dots (i) \quad 1$$

this plane also passes through points.

(6, 4, -5) and (-4, -2, 3)

$$5a + 3b + 4c = 0 \quad 1$$

$$\text{and } -5a - 3b + 4c = 0 \text{ or} \quad 1$$

$$5a + 3b - 4c = \dots\dots\dots (iii)$$

Equation (ii) and (iii) are same. 1

∴ a, b, c have number of infinite solutions. 1

Hence eq(i) represent infinite planes. 1

20. For graphical representation 2

For Tabulation 2

Max. z = 6250 1

at x = 10, y = 50 1

4381/4331/(Set : A, B, C & D)