SET – D

SECTION – A

1. (i) **Ans. (A)**

$\because$ f is one-one and onto

$\because 3-4x_1 = 3-4x_2 \Rightarrow x_1 = x_2$

and $y = 3-4x \Rightarrow x = \dfrac{3-y}{4} \in R$ 1

(ii) **Ans. (C)**

$$\because \cot^{-1}\left(-\frac{1}{\sqrt{3}}\right) = -\cot^{-1}\left(\frac{1}{\sqrt{3}}\right) = -\cot^{-1}\left(\cos\frac{\pi}{3}\right)$$

$$= \cos^{-1}\cos\left(\pi-\frac{\pi}{3}\right) = \frac{2\pi}{3}$$ 1

(iii) **Ans. (B)**

$\because\ 2 + y = 5 \Rightarrow y = 3$ and $2x + 2 = 8 \Rightarrow x = 3$ 1

(iv) **Ans. (D)**

$\because |kA| = k^3\ |A|$ 1

(v) **Ans. (B)**

$$\underset{x\to\pi^-}{Lt} f(x) = \underset{x\to\pi^+}{Lt} f(x) \Rightarrow \underset{x\to\pi^-}{Lt} kx+1$$

$$= \underset{x\to\pi^+}{Lt} \sin x \Rightarrow k\pi+1 = \sin\pi = 0 \Rightarrow k = -\frac{1}{\pi}$$ 1

(vi) **Ans. (A)**

$$A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r = 2\pi \,.\, 3 = 6\pi \text{ cm}^2/\text{sec}$$ 1

(vii) **Ans. (C)**

$$\frac{2x}{25} + \frac{2y}{4} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{25} \times \frac{4}{2y} = 0 \Rightarrow \therefore x = 0$$

$\therefore\ y = \pm 2$ 1

(viii) **Ans. (D)**

Put $x^2 = t \Rightarrow 2x\,dx = dt$

$$\because \frac{1}{2}\int \frac{2x\,dx}{\sqrt{a^4 - (x^2)^2}} = \frac{1}{2}\int \frac{dt}{\sqrt{a^4 - t^2}} = \frac{1}{2}\sin^{-1}\frac{t}{a^2} + c$$

$$= \frac{1}{2}\sin^{-1}\frac{x^2}{a^2} + c$$ 1

(ix) **Ans. (A)**

$$\because \int_0^1 xe^x dx = x \cdot e^x - \int 1 \cdot e^x dx = xe^x - e^x = (x-1)e^x \Big|_0^1$$

$$= (1-1)e^1 - (0-1)0 \Rightarrow 1$$ 1

(x) **Ans. (A)**

$$\because x^2 + y^2 = 2cx \Rightarrow 2x + 2y + \frac{dy}{dx} = 2c \Rightarrow c = x + y \cdot \frac{dy}{dx}$$

$$\therefore \text{ Diff. equ. is } x^2 + y^2 = 2x\left(x + y\frac{dy}{dx}\right)$$

$$\Rightarrow 2xy\frac{dy}{dx} + x^2 - y^2 = 0$$ 1

(xi) **Ans. (A)**

$$\left(1+x^2\right)\frac{dy}{dx} = 1 + y^2 \Rightarrow \frac{dy}{1+y^2} = \frac{-dx}{1+x^2} \text{ integrating,}$$

$$\tan^{-1} y = \tan^{-1} x + c \Rightarrow \tan^{-1} x + \tan^{-1} y = c$$

1

(xii) **Ans. (C)**

Projection of $\vec{a} = \hat{i} - 2\hat{j} + \hat{k}$ on $\vec{b} = 4\hat{i} - 4\hat{j} + 7\hat{k}$

$$\frac{\vec{a} \cdot \vec{b}}{|\vec{b}|} = \frac{1.4 + (-2)(-4) + 1.7}{\sqrt{4^2 + 4^2 + 7^2}} = \frac{19}{9}$$ 1

(xiii) **Ans. (D)**

$\therefore\ (-3)(3k)+2k.1+2(-5)=0 \Rightarrow -9k+2k-10=0$

$\Rightarrow -7k=10 \Rightarrow k=-\dfrac{10}{7}$ 1

(xiv) **Ans. (B)**

No. of outcomes = 36

A total of 8 of two dice may be (2, 6); (3, 5); (4, 4); (5, 3); (6, 2)

$\therefore$ No. of favourable cases = 5 1

$\therefore$ Probability of getting a total of 8 = $\dfrac{5}{36}$

(xv) **Ans. (B)**

$\because P(B/A)=\dfrac{P(A\cap B)}{P(A)}=\dfrac{P(A\cap B)}{0}$

which is not defined 1

(xvi) **Ans. (D)**

Here n = 8 and $p=\dfrac{1}{2}$

$\therefore P(X=4)={}^{8}c_4\left(\dfrac{1}{2}\right)^4\left(\dfrac{1}{2}\right)^4=\dfrac{\underline{|8}}{\underline{|4}\ \underline{|4}}\ \dfrac{1}{16\times 16}$

$=\dfrac{8.7.6.5}{16\times 16\times 14}=\dfrac{35}{128}$ 1

SECTION – B

2. $f(x) = \dfrac{4x+3}{6x-4} = y$ (say)

$$\therefore\ fof(x) = f(f(x)) = f(y) = \frac{4y+3}{6y-4} = \frac{4\,.\,\dfrac{4x+3}{6x-4}+3}{6\,\dfrac{4x+3}{6x-4}-4}$$

$$= \frac{16x+12+18x-12}{24x+18-24x+16} = \frac{34x}{34} = x \qquad \text{(proved)} \qquad 2$$

3. L. H. S.

$$\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{2}{11} = \tan^{-1}\left(\frac{\dfrac{1}{2}+\dfrac{2}{\pi}}{1-\dfrac{1}{2}\,.\,\dfrac{2}{\pi}}\right) = \tan^{-1}\left(\frac{11+4}{22-2}\right)$$

$$= \tan^{-1}\left(\frac{15}{20}\right) = \tan^{-1}\left(\frac{3}{4}\right) = \text{R. H. S.} \qquad 2$$

4. Let $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} \therefore A = AI \Rightarrow \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = A\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Operating $c_1 \leftrightarrow c_2 \Rightarrow \begin{bmatrix} -1 & 3 \\ 2 & -4 \end{bmatrix} = A\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$

Operating c_1

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ -2 & -4 \end{bmatrix} = A \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ -2 & 2 \end{bmatrix} = A \begin{bmatrix} 0 & 1 \\ -1 & +3 \end{bmatrix}$$

Operating $(c_2 - 3c_1)$ Operating $c_1 - c_2$ 1

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = A \begin{bmatrix} 1 & 1 \\ +2 & 3 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 3 \end{bmatrix}$$

Operating $-\frac{1}{2} c_2$

$$\therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & \frac{3}{2} \end{bmatrix}$$ 1

5. $\begin{vmatrix} 3 & 1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ Expanding by R_2, we get

$$= (-1)^{2+3}(-1) \begin{vmatrix} 3 & 1 \\ 3 & -5 \end{vmatrix} = -15 - 3 = -18$$ 2

6. Let $I = \int \frac{dx}{9x^2 - 12x + 8} = \frac{1}{9} \int \frac{dx}{x^2 - \frac{4}{3}x + \frac{8}{9}}$

$$= \frac{1}{9} \int \frac{dx}{x^2 - \frac{4}{3}x + \frac{4}{9} + \frac{8}{9} - \frac{4}{9}}$$

$$= \frac{1}{9}\int \frac{dx}{\left(x-\frac{2}{3}\right)^2+\frac{4}{9}} \quad \text{Put } x-\frac{2}{3}=t \Rightarrow dx=dt$$ 1

$$= \frac{1}{9}\int \frac{dt}{t^2+\left(\frac{2}{3}\right)^2} = \frac{1}{9}\cdot\frac{1}{\frac{2}{3}}\tan^{-1}\frac{t}{2/3}+c$$

$$= \frac{1}{9}\times\frac{3}{2}\tan^{-1}\frac{\left(x-\frac{2}{3}\right)^3}{2}+c$$

$$= \frac{1}{6}\tan^{-1}\left(\frac{3x-2}{2}\right)+c$$ 1

7. Let $I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x+\cos^4 x}\,dx$ .... (i)

Using the property $= \int_0^a f(x)\,dx = \int_0^a f(a-x)\,dx$

$$\therefore I = \int_0^{\pi/2} \frac{\sin^4\left(\frac{\pi}{2}-x\right)}{\sin^4\left(\frac{\pi}{2}-x\right)+\cos^4\left(\frac{\pi}{2}-x\right)}\,dx$$

$$= \int_0^{\pi/2} \frac{\cos^4 x}{\cos^4 x+\sin^4 x}\,dx \text{ .... (ii)}$$ 1

Adding (i) and (ii) we get

$$I + I = \int_0^{\pi/2} \frac{\sin^4 x}{\sin^4 x + \cos^4 x}\, dx + \int_0^{\pi/2} \frac{\cos^4 x dx}{\sin^4 x + \cos^4 x}$$

$$= \int_0^{\pi/2} 1\, dx = \frac{\pi}{2}$$

$\therefore I = \frac{\pi}{4}$ 1

8. $xy = ae^x + be^{-x}$ Diff. w. r. t. x, we get

$1.y + x.\frac{dy}{dx} = ae^x - be^{-x}$ Again Diff w. r. t. x, we get 1

$$\frac{dy}{dx} + 1.\frac{dy}{dx} + x.\frac{d^2y}{dx^2} = ae^x + be^{-x} = xy$$

$$\Rightarrow x.\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - xy = 0$$

is the required diff. equation 1

9. The diff. equation is $y^2.\frac{dy}{dx} + y^2 + 1 = 0$

$\Rightarrow y^2 \frac{dy}{dx} = -\left(y^2 + 1\right)$ separating the variables

$\Rightarrow \frac{y^2}{y^2 + 1} dy = -dy$ Integrating

$$\Rightarrow \int \frac{y^2}{y^2+1} dy = -\int dx + c = -x + c$$ 1

$$\Rightarrow \int \frac{y^2+1-1}{y^2+1} dy = \int \left(1 - \frac{1}{y^2+1}\right) dy = -x + c$$

$$\Rightarrow y - \tan^{-1} y = -x + c \Rightarrow x + y = \tan^{-1} y + c$$ 1

10. Let $u = \tan^2 x$ and $v = \sec^2(x^2)$

$$\therefore \frac{du}{dx} = 2\tan x . \sec^2 x \text{ and}$$

$$\frac{dv}{dx} = 2\sec(x^2) . \sec x^2 \tan x^2 \; . \; 2x$$ 1

$$\therefore \frac{du}{dv} = \frac{\frac{du}{dx}}{\frac{dv}{dx}} = \frac{2\tan x \sec^2 x}{4x \sec^2(x^2) \tan(x^2)}$$

$$= \frac{\tan x \sec^2 x}{2x \sec^2(x^2) \tan(x^2)}$$ 1

11. Let A be the event of getting 4, 5 or 6 on the first toss and B the event of getting 1, 2, 3, or 4 on the 2nd toss. Let S be the sample space when a die is thrown.

i.e. S = {1, 2, 3, 4, 5, 6}

$\therefore$ Probability of getting 4, 5 or 6 on the first toss $= P(A) = \frac{3}{6} = \frac{1}{2}$ and probability of getting 1, 2, 3, or 4 on the 2nd toss $= P(B) = \frac{4}{6} = \frac{2}{3}$ 1

Now the events A and B are independent

$\therefore$ Probability of getting 4, 5, or 6 on first toss and 1, 2, 3 or 4 on the 2nd toss

$= P(AB) = P(A).P(B) = \frac{1}{2} \times \frac{2}{3} = \frac{1}{3}$ 1

SECTION – C

12. Let $\tan^{-1}\sqrt{x} = y \Rightarrow \tan y = \sqrt{x} \Rightarrow x = \tan^2 y$

Since $\cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta} \therefore \cos 2y = \frac{1-\tan^2 y}{1+\tan^2 y}$ 1

$\Rightarrow \cos 2y = \frac{1-x}{1+x} \therefore 2y = \cos^{-1}\left(\frac{1-x}{1+x}\right)$

$\Rightarrow y = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right) \Rightarrow \tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

3

13. $x^y = y^x$ Taking logarithm on both sides, we get

$y \log x = x \log y$ 1

Diff. w. r. t. x

$$y \cdot \frac{1}{x} + \log x \cdot \frac{dy}{dx} = 1 \cdot \log y + x \cdot \frac{1}{y}\frac{dy}{dx}$$

$$\Rightarrow \left(\log x - \frac{x}{y}\right)\frac{dy}{dx} = \log y - \frac{y}{x} = \frac{x\log y - y}{x}$$

$$\Rightarrow \frac{dy}{dx} = \frac{x\log y - y}{x\left(\frac{y\log x - x}{y}\right)} = \frac{y(x\log y - y)}{x(y\log x - x)}$$ 3

14. $f(x) = 3x^4 - 2x^3 - 6x^2 + 6x + 1 \quad x \in [0, 2]$

$f'(x) = 12x^3 - 6x^2 - 12x + 6 = 0$

$\Rightarrow 2x^3 - x^2 - 2x + 1 = 0 \Rightarrow (2x^3 - 2x) - (x^2 - 1) = 0$

$\Rightarrow 2x(x^2 - 1) - (x^2 - 1) = 0 \Rightarrow (x^2 - 1) - (2x - 1) = 0$

$\Rightarrow x = \pm 1$ and $x = \frac{1}{2}$ But $-1 \notin [0, 2]$

$\therefore x = 1$ and $\frac{1}{2}$ 2

$\therefore f(0) = 1, f(1) = 2,$

$$f\left(\frac{1}{2}\right) = 3 \cdot \frac{1}{16} - 2 \cdot \frac{1}{8} - 6\frac{1}{4} + 6 \cdot \frac{1}{2} + 1 = \frac{23}{16}$$

$f(2) = 3.2^4 - 2.2^3 - 6.2^2 + 6.2 + 1$

= 48 – 16 – 24 + 12 + 1 = 21

$\therefore$ Absolute max. value = 21 and absolute minimum = 1 2

15. Let E be the event that A throws a head and F be the event that B throws a head.

$\therefore P(E) = \frac{1}{2}$ and $P(F) = \frac{1}{2}$ $\therefore P(\overline{E}) = 1 - P(E) = \frac{1}{2}$ 1

and $P(\overline{F}) = 1 - P(F) = \frac{1}{2}$

Now A wins if he throws head in 1st, 3rd, 5th .... throws and B does not throw head in between these throws.

$\therefore$ Probability that A wins = P [E or $\overline{E}\overline{F}E$ or $\overline{E}\overline{F}\overline{E}\overline{F}E$ = ...

= P(E) + $P(\overline{E}\overline{F}E) + P(\overline{E}\overline{F}\overline{E}\overline{F}E) + \ldots\ldots$

= P(E)+ $P(\overline{E})\, P(\overline{F})$.P(E)+ $P(\overline{E})\, P(\overline{F})\;\; P(\overline{E})\, P(\overline{F})$. P(E)

$= \frac{1}{2} + \left(\frac{1}{2}\right)^3 + \left(\frac{1}{2}\right)^5$

$= \frac{\frac{1}{2}}{1 - \left(\frac{1}{2}\right)^2} = \frac{1}{2} \times \frac{4}{3} = \frac{2}{3}$ 3

16. Let $\vec{a}$, $\vec{b}$, $\vec{c}$ be the position vectors of the points A, B, C respectively

$\therefore\ \vec{a} = 2\hat{i} - \hat{j} + \hat{k};\ \vec{b} = \hat{i} - 3\hat{j} - 5\hat{k}$ and $\vec{c} = 3\hat{i} - 4\hat{j} - 4\hat{k}$

$\therefore \overrightarrow{AB} = P.V.\text{ of } B - P.V.\text{ of } A = \hat{i} - 3\hat{j} - 5\hat{k} - \left(2\hat{i} - \hat{j} + \hat{k}\right)$

$$= -\hat{i} - 2\hat{j} - 6\hat{k}$$

$\overrightarrow{BC} = 3\hat{i} - 4\hat{j} - 4\hat{k} - \left(\hat{i} - 3\hat{j} - 5\hat{k}\right) = 2\hat{i} + \hat{j} + \hat{k}$

$\overrightarrow{AC} = 2\hat{i} - \hat{j} + \hat{k} - \left(3\hat{i} - 4\hat{j} - 4\hat{k}\right) = -\hat{i} + 3\hat{j} + 5\hat{k}$

$\therefore \overrightarrow{AB} + \overrightarrow{BC} + \overrightarrow{CA} = 0 \Rightarrow ABC$ is a triangle. 2

Now $|\overrightarrow{AB}| = \sqrt{(-1)^2 + (-2)^2 + (-6)^2} = \sqrt{41}$

$|\overrightarrow{BC}| = \sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6}$

$|\overrightarrow{CA}| = \sqrt{(-1)^2 + (3)^2 + (5)^2} = \sqrt{35}$

Now $BC^2 + CA^2 = 35 + 6 = 41 = AB^2$

$\therefore\ ABC$ is a right triangle 2

SECTION – D

17. The system of linear eqs. can be written as $AX = B$ where $A = \begin{bmatrix} 3 & 1 & 2 \\ 2 & -3 & -1 \\ 1 & 2 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix}$

$|A| = 3(-3 + 2) - 1(2 + 1) + 2(4 + 3) = 8 \neq 0$

$\therefore$ A^{-1} exists 1

To find A^{-1}, $A_{11} = -1$, $A_{12} = -3$, $A_{13} = 7$

$A_{21} = 3$, $A_{22} = 1$, $A_{23} = -5$

$A_{31} = 5$, $A_{32} = 7$, $A_{33} = -11$

$$\therefore A^{-1} = \frac{adjA}{|A|} = \frac{1}{8}\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{8}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}$$ 3

$$\therefore X = A^{-1}B = \frac{1}{2}\begin{bmatrix} -1 & 3 & 5 \\ -3 & 1 & 7 \\ 7 & -5 & -11 \end{bmatrix}\begin{bmatrix} 3 \\ -3 \\ 4 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} -3-9+20 \\ -9-3+28 \\ 21+15-44 \end{bmatrix}$$

$= \frac{1}{8}\begin{bmatrix} 8 \\ 16 \\ -8 \end{bmatrix}$ $\therefore$ $x = 1$, $y = 2$, $z = -1$ 2

18. The given curves are $y = x + 2$ and $y = \frac{1}{3}x^2 + 2$

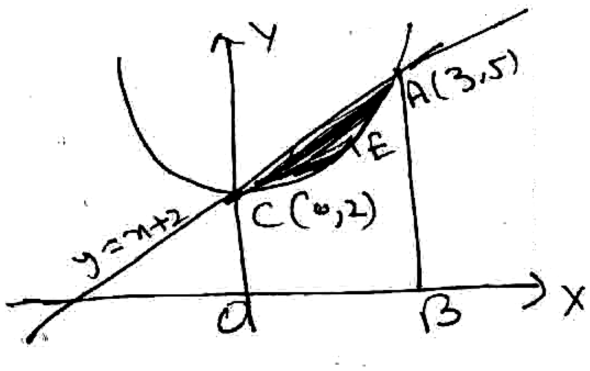
To find the points of intersection of the two curves

Solve them, $x + 2 = \frac{1}{3}x^2 + 2 \Rightarrow x = \frac{x^3}{3} \Rightarrow x^2 - 3x = 0$

$\Rightarrow x = 0$ or $x = 3 \therefore y = 2$ or $y = 5$

Thus the points of intersection of the given curves are (0, 2) and (3, 5)

Now $y = \frac{1}{3}x^2 + 2$ is a parabola with vertex at (0, 2). It is symmetrical to y-axis as it contains even powers of x. Also $y = x + 2$ is a st. line passing through (0, 2) and (3, 5).



3

$\therefore$ Required Area = area $CEAC$

$$= \text{area } (OCAB) - \text{area } (OCEAB)$$

$$= \int_0^3 y \text{ of line } dx - \int_0^3 y \text{ of parabola } dx$$

$$= \int_0^3 (x+2)\, dx - \int_0^3 \left(\frac{1}{3}x^2 + 2\right) dx$$

$$= \left(\frac{x^2}{2} + 2x\right)_0^3 - \left(\frac{1}{3}\,\frac{x^3}{3} + 2x\right)\Bigg|_0^3$$

$$= \frac{21}{2} - (3+6) = \frac{21}{2} - 9 = \frac{3}{2} \text{ sq. units}$$

3

OR

The given curves are $y = x^2 + 5$ (i)

and $y = x^3$

For $1 \le x \le 2,\ \ y = x^2 + 5 \ge 0$

$\therefore$ Graph of the curve $y = x^2 + 5$ lies above x-axis

for $1 \le x \le 2$; $y = x^3 \ge 0$

$\therefore$ graph of the curve $y = x^3$ also lies above x-axis

for $1 \le x \le 2$; $x^2 + 5 > x^2$ 3

$\therefore$ Required area $= \int_1^2 \left(y_{upper} - y_{lower}\right) dx$ 3

$$= \int_1^2 \left(x^2 + 5 - x^3\right) dx = \left.\frac{x^3}{3} + 5x - \frac{x^4}{4}\right|_1^2$$

$$= \left(\frac{8}{3} + 10 - 4\right) - \left(\frac{1}{3} + 5 - \frac{1}{4}\right)$$

$$= \frac{26}{3} - \frac{61}{12} = \frac{43}{12} \text{ sq. units}$$ 3

19. The given lines are $\vec{r} = \hat{i} + 2\hat{j} + \hat{k} + \lambda\left(\hat{i} - \hat{j} + \hat{k}\right)$ ... (i)

and $\vec{r} = 2\hat{i} - \hat{j} - \hat{k} + \mu\left(2\hat{i} + \hat{j} + 2\hat{k}\right)$ .... (ii)

Comparing the given lines with $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and

$\vec{r} = \vec{a_2} + \lambda\vec{b_2}$

$\therefore\ \vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b_1} = \hat{i} - \hat{j} + \hat{k}$

and $\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$

$\therefore\ \vec{a_2} - \vec{a_1} = \left(2\hat{i} - \hat{j} - \hat{k}\right) - \left(\hat{i} + 2\hat{j} + \hat{k}\right) = \hat{i} - 3\hat{j} - 2\hat{k}$ 2

and $\vec{b_1} \times \vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = \hat{i}(-2-1) - \hat{j}(2-2) + \hat{k}(1+2)$

$$= -3\hat{i} + 3\hat{k}$$

$\therefore\ |\vec{b_1} \times \vec{b_2}| = \sqrt{(-3)^2 + (3)^2} = 3\sqrt{2}$

$\left(\vec{a_2} - \vec{a_1}\right) . \left(\vec{b_1} \times \vec{b_2}\right) = \left(\hat{i} - 3\hat{j} - 2\hat{k}\right) . \left(-3\hat{i} + 3\hat{k}\right)$

$$= -3 - 6 = -9$$

$\therefore$ S. D. between the lines (i) and (ii)

$$= \left| \frac{\left(\vec{a}_2 - \vec{a}_1\right) \times \left(\vec{b}_1 \times \vec{b}_2\right)}{|\vec{b}_1 \times \vec{b}_2|} \right|$$

$$= \frac{|-9|}{3\sqrt{2}} = \frac{3}{\sqrt{2}} = \frac{3}{2}\sqrt{2} \text{ sq. units}$$ 4

OR

The equation of plane passing through the line of intersection of planes $\vec{r}\,.\left(2\hat{i} - 3\hat{j} + 4\hat{k}\right) - 1 = 0$ and $\vec{r}\,.\left(\hat{i} - \hat{j}\right) + 4 = 0$ is

$$\vec{r}\,.(2\hat{i} - 3\hat{j} + 4\hat{k}) - 1 + \lambda\left(\vec{r}\,.(\hat{i} - \hat{j}) + 4\right) = 0$$

$$\Rightarrow \vec{r}\,.\left((2+\lambda)\hat{i} - (3+\lambda)\hat{j} + 4\hat{k} - 1 + 4\lambda\right) = 0$$ 2

Since (i) is perpendicular to the plane

$\therefore \vec{r}\,.(2\hat{i} - \hat{j} + \hat{k}) + 8 = 0$

$$\therefore (2+\lambda)2-(3+\lambda)(-1)+4.1=0$$

$$\Rightarrow 4+2\lambda+3+\lambda+4=0 \Rightarrow \lambda=-\frac{11}{3}$$ 2

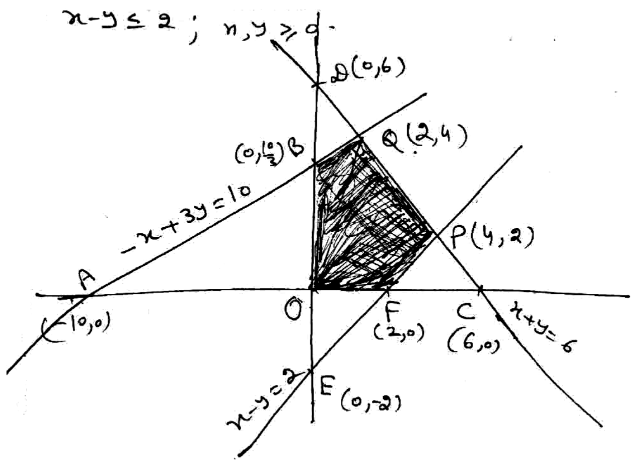
Putting in (ii), we get

$$\vec{r}\,.\left(\left(2-\frac{11}{3}\right)\hat{i}-\left(3-\frac{11}{3}\right)\hat{j}+4\hat{k}-1+4\left(-\frac{11}{3}\right)\right)=0$$

$$\vec{r}\,.\left(-\frac{5}{3}\hat{i}+\frac{2}{3}\hat{j}+4\hat{k}\right)-\frac{47}{3}=0$$

$$\vec{r}\,.(-5\hat{i}+2\hat{j}+12\hat{k})=47$$ 2

20. Draw the graph of $-x+3y\le 10;\ x+y\le 6$



3

The co-ordinates of extreme points of the feasible region are $O(0, 0)$; $F(2, 0)$; $P(4, 2)$; $Q(2, 4)$; $B\left(0, \frac{10}{3}\right)$ Now $z = x + 2y$

The value of z at $(0, 0) = 0$

at $F(2, 0) = 2$

at $P(4, 2) = 8$

at $Q(2, 4) = 10$

at $B\left(0, \frac{10}{3}\right) = \frac{20}{3}$

$\therefore$ Maximum value of z is 10 at $Q(2, 4)$ i.e. $x = 2$, $y = 4$. 3

CLASS : 12th (Sr. Secondary) **2081/2031**

Series : SS-M/2017

Total No. of Printed Pages : **78** ***SET : A, B, C & D***

MARKING INSTRUCTIONS AND MODEL ANSWERS

MATHEMATICS

ACADEMIC/OPEN

(Only for Fresh Candidates)

उप-परीक्षक मूल्यांकन निर्देशों का ध्यानपूर्वक अवलोकन करके उत्तर. पुस्तिकाओं का मूल्यांकन करें। यदि परीक्षार्थी ने प्रश्न पूर्ण व सही हल किया है तो उसके पूर्ण अंक दें।

General Instructions :

(i) *Examiners are advised to go through the general as well as specific instructions before taking up evaluation of the answer-books.*

(ii) *Instructions given in the marking scheme are to be followed strictly so that there may be uniformity in evaluation.*

(iii) *Mistakes in the answers are to be underlined or encircled.*

(iv) *Examiners need not hesitate in awarding full marks to the examinee if the answer/s is/are absolutely correct.*

(v) *Examiners are requested to ensure that every answer is seriously and honestly gone through before it is awarded mark/s. It will ensure the authenticity as their evaluation and enhance the reputation of the Institution.*

(vi) *A question having parts is to be evaluated and awarded partwise.*

(vii) *If an examinee writes an acceptable answer which is not given in the marking scheme, he or she may be awarded marks only after consultation with the head-examiner.*

(viii) *If an examinee attempts an extra question, that answer deserving higher award should be retained and the other scored out.*

(ix) *Word limit wherever prescribed, if violated upto 10%. On both sides, may be ignored. If the violation exceeds 10%, 1 mark may be deducted.*

(x) *Head-examiners will approve the standard of marking of the examiners under them only after ensuring the non-violation of the instructions given in the marking scheme.*

(xi) Head-examiners and examiners are once again requested and advised to ensure the authenticity of their evaluation by going through the answers seriously, sincerely and honestly. The advice, if not headed to, will bring a bad name to them and the Institution.

महत्त्वपूर्ण निर्देश :

(i) अंक-योजना का उद्देश्य मूल्यांकन को अधिकाधिक वस्तुनिष्ठ बनाना है। अंक-योजना में दिए गए उत्तर-बिन्दु अंतिम नहीं हैं। ये सुझावात्मक एवं सांकेतिक हैं। यदि परीक्षार्थी ने इनसे भिन्न, किन्तु उपयुक्त उत्तर दिए हैं, तो उसे उपयुक्त अंक दिए जाएँ।

(ii) शुद्ध, सार्थक एवं सटीक उत्तरों को यथायोग्य अधिमान दिए जाएँ।

(iii) परीक्षार्थी द्वारा अपेक्षा के अनुरूप सही उत्तर लिखने पर उसे पूर्णांक दिए जाएँ।

(iv) वर्तनीगत अशुद्धियों एवं विषयांतर की स्थिति में अधिक अंक देकर प्रोत्साहित न करें।

(v) भाषा-क्षमता एवं अभिव्यक्ति-कौशल पर ध्यान दिया जाए।

(vi) *मुख्य-परीक्षकों / उप-परीक्षकों को उत्तर-पुस्तिकाओं का मूल्यांकन करने के लिए केवल Marking Instructions/ Guidelines दी जा रही हैं, यदि मूल्यांकन निर्देश में किसी प्रकार की त्रुटि हो, प्रश्न का उत्तर स्पष्ट न हो, मूल्यांकन निर्देश में दिए गए उत्तर से अलग कोई और भी उत्तर सही हो तो परीक्षक, मुख्य-परीक्षक से विचार-विमर्श करके उस प्रश्न का मूल्यांकन अपने विवेक अनुसार करें।*

SET – A

SECTION – A

1. (i) **Ans. (D)**

$\because x_1^4 = x_2^4 \Rightarrow x_1^2 = \pm x_2^2 \Rightarrow x_1 = \pm x_2$

$\therefore f$ is not $1 - 1$

$\therefore f(-1) = f(-1)$ but $-1 \neq 1$. Also -2 in the co-domain is not image of any element x in the domain R

$\therefore f$ is not onto 1

(ii) **Ans. (B)**

$$\because \tan^{-1}\sqrt{3} - \sec^{-1}(-2) = \frac{\pi}{3} - \left(\pi - \frac{\pi}{3}\right)$$

$$= \frac{\pi}{3} - \frac{2\pi}{3} = -\frac{\pi}{3}$$ 1

(iii) **Ans. (C)**

$\because\ 3x + 7 = 5 \Rightarrow x = -\dfrac{2}{3}$ and $5 = y - 2 \Rightarrow y = 7$ 1

(iv) **Ans. (D)**

$\because\ |\text{adj } A\ .\ A| = |A|^3 \Rightarrow |\text{adj } A| = |A|^2$ 1

(v) **Ans. (B)**

$$\underset{x\to\pi^-}{Lt}\ kx + 1 = \underset{x\to\pi^+}{Lt}\ \cos x \Rightarrow k\,\pi + 1 = \cos\pi = -1$$

$$\Rightarrow k = -\frac{2}{\pi}$$ 1

(vi) **Ans. (A)**

Area $= A = \pi r^2 \Rightarrow \dfrac{dA}{dr} = 2\pi r = 2\pi 5 = 10\pi$ 1

(vii) **Ans. (D)**

$\dfrac{dy}{dx} = 3x^2 - 11$ slope of $y = x - 11$ is 1

$\therefore 3x^2 - 11 = 1 \Rightarrow x = \pm 2 \therefore y = -9$ 1

(viii) **Ans. (A)**

Put $x^3 = t \Rightarrow 3x^2 dx = dt$

$$\therefore \int \frac{x^2}{x^6 + 1} dx = \frac{1}{3}\int \frac{dt}{t^2 + 1}$$

$$= \frac{1}{3}\tan^{-1} t + c = \frac{1}{3}\tan^{-1} x^3 + c$$ 1

(ix) **Ans. (B)**

$$\int_0^1 \sin^{-1} x dx = x \,.\, \sin^{-1} x - \int \frac{1}{\sqrt{1-x^2}} \,.\, x dx$$

$$= x \sin^{-1} x - \frac{1}{2} \frac{\left(1-x^2\right)^{-\frac{1}{2}+1}}{\frac{1}{2}+1}$$

$$= \sin^{-1} 1 - \frac{1}{2} \frac{(1-0)^{\frac{1}{2}}}{\frac{1}{2}} = \frac{\pi}{2} - 1 \qquad 1$$

(x) **Ans. (C)**

$$\because y = c_1 e^x + c_2 e^{-x} \Rightarrow \frac{dy}{dx} = c_1 e^x - c_2 e^{-x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = c_1 e^x + c_2 e^{-x} = y \quad 1$$

(xi) **Ans. (A)**

$$\because \frac{dy}{dx} = -y - 2xe^{-x} \Rightarrow \frac{dy}{dx} + y = -2xe^{-x} \quad \therefore P = 1$$

I. F. e^x $\qquad$ $Q = -2xe^{-x}$

$\therefore$ Solution is $y.e^x = -\int e^x \,.\, 2xe^{-x} dx + c$ 1

$= -x^2 + c$

$ye^x + x^2 = c$

(xii) **Ans. (D)**

Projection of $\vec{a}$ on $\vec{b} = \dfrac{|\vec{a}\,.\,\vec{b}|}{|\vec{b}|} = \dfrac{2.1+3.2+2.1}{\sqrt{6}}$

$$= \frac{10}{\sqrt{6}} = \frac{5}{3}\sqrt{6} \qquad 1$$

(xiii) **Ans. (A)**

$$\therefore \cos\theta = \frac{3.1+5.1+4.2}{\sqrt{3^2+5^2+4^2}\sqrt{1^2+1^2+2^2}}$$

$$= \frac{16}{\sqrt{50}\sqrt{6}} = \frac{16}{5\sqrt{2}\sqrt{6}} = \frac{8\sqrt{3}}{15} \qquad 1$$

(xiv) **Ans. (B)**

No. of outcomes = 36 and even prime number is 2

A = possible outcomes = 1 1

$$\therefore P(A) = \frac{1}{36}$$

(xv) **Ans. (C)**

$$p(X=6) = {}^{10}C_6\left(\frac{1}{2}\right)^{10} = \frac{\underline{|10}}{\underline{|6}\ \ \underline{|4}} \cdot \left(\frac{1}{2}\right)^{10} \qquad 1$$

$$= \frac{105}{512}$$

Here $n = 10,\ p = \dfrac{1}{2}$

(xvi) **Ans. (D)**

$$\because P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0}$$

which not defined 1

SECTION – B

2. Let $y = 4x + 3 \Rightarrow \quad x = \dfrac{y-3}{4}$ Let $f(x) = 4x + 3$ and

$g(x) = x = \dfrac{y-3}{4}$ 1

$$\therefore \; fog(x) = f\left(\frac{x-3}{4}\right) = 4 \,.\left(\frac{x-3}{4}\right) + 3 = x$$

$$gof(x) = g(4x+3) = \frac{4x+3-3}{4} = x$$

Thus $fog = gof = I$

$\therefore$ g is the inverse of f and f is invertible

$g : Y \rightarrow N$ and $g(x) = \dfrac{x-3}{4}$ 1

3. $\because 2\sin^{-1} x = \sin^{-1}\left(2x\sqrt{1-x^2}\right) \Rightarrow 2\sin^{-1}\dfrac{3}{5}$

$$= \sin^{-1}\left(2 \,.\, \frac{3}{5}\sqrt{1\frac{-9}{25}}\right)$$ 1

$$\Rightarrow 2\sin^{-1}\frac{3}{5} = \sin^{-1}\frac{24}{25} = \tan^{-1}\left(\frac{x}{1-x^2}\right)$$

$$= \tan^{-1}\left(\frac{\frac{24}{25}}{\sqrt{1-\left(\frac{24}{25}\right)^2}}\right)$$

$$= \tan^{-1}\frac{24}{25}\times\frac{25}{7} = \tan^{-1}\frac{24}{7} = \text{R. H. S.}$$ 1

4. $A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$\therefore A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix}$

$\therefore (A + A')' = \begin{bmatrix} 2 & 22 \\ 11 & 14 \end{bmatrix} = A + A'$

$\therefore$ $A + A'$ is a symmetric 2

5. Expanding along R_1, we get

$$0\begin{vmatrix} 0 & \sin\beta \\ -\sin\beta & 0 \end{vmatrix} - \sin\alpha\begin{vmatrix} \sin\alpha & \sin\beta \\ \cos\alpha & 0 \end{vmatrix} - \cos\alpha\begin{vmatrix} -\sin\alpha & 0 \\ \cos\alpha & -\sin\beta \end{vmatrix}$$

$= -\sin\alpha\,(-\sin\beta\cos\alpha) - \cos\alpha\,(\sin\alpha\sin\beta)$

$= \sin\alpha\sin\beta\cos\alpha - \sin\alpha\sin\beta\cos\alpha = 0$ 2

6. $\int \sqrt{x^2+2x+5}\, dx = \int \sqrt{(x+1)^2+4}dx$

Put $x + 1 = t \Rightarrow dx = dt$

$$= \int \sqrt{t^2+(2)^2}\, dt = \frac{1}{2}\, t\sqrt{t^2+4} + \frac{4}{2}\log\left| t+\sqrt{t^2+4} \right| + c$$

1

$$= \frac{x+1}{2}\sqrt{x^2+2x+5} + 2\log\left| x+1+\sqrt{x^2+2x+5} \right| + C$$

1

7. Let $f(x) = \sin^5 x \cos^4 x$

Then $f(-x) = \sin^5(-x)\cos^4(-x)$

$= \sin^5 x \cos^4 x = -f(x)$ 1

$\therefore f$ is an odd function. Hence $\int_{-1}^{1} f(x)\, dx = 0$ 1

8. Equation of the circle is $(x-a)^2 + (y-b)^2 = r^2$

Since the family of circles touch y-axis at the origin

$\therefore$ Centre lies on x-axis Hence $b = 0$ 1

$\therefore$ Equation of circle is $(x-a)^2 + y^2 = a^2$

Diff. w.r.t. x, $2(x-a) + 2y\dfrac{dy}{dx} = 0 \Rightarrow x - a = -y \,.\, \dfrac{dy}{dx}$

$\therefore$ Its diff. eq. is $\left(-y \,.\, \dfrac{dy}{dx}\right)^2 + y^2 = a^2$

$\Rightarrow y^2 \,.\, \left(\dfrac{dy}{dx}\right)^2 + y^2 = a^2$ 1

9. $y = A \sin x + B \cos x \Rightarrow \dfrac{dy}{dx} = A \cos x - B \sin x$

Again diff.

$\Rightarrow \dfrac{d^2y}{dx^2} = -A \sin x - B \cos x$

$= -(A \sin x + B \cos x) = -y$

$\Rightarrow \dfrac{d^2y}{dx^2} + y = 0$ 2

10. Let $u = \sin^2 x$ and $v = e^{\cos x}$

$\therefore \dfrac{du}{dx} = 2 \sin x \cos x$ and $\dfrac{dv}{dx} = e^{\cos(x)}(-\sin x)$ 1

$\therefore \dfrac{du}{dv} = \dfrac{\frac{du}{dx}}{\frac{dv}{dx}} = \dfrac{2 \sin x \cos x}{-\sin x \,.\, e^{\cos x}} = -2 \cos x \,.\, e^{-\cos x}$ 1

11. We have $P(A) = \dfrac{18}{36} = \dfrac{1}{2}$ and $P(B) = \dfrac{18}{36} = \dfrac{1}{2}$

Also $P(A \cap B)$ = P(odd number on both throws)

$$= \frac{9}{36} = \frac{1}{4}$$ 1

Now $P(A) \,.\, P(B) = \dfrac{1}{2} \,.\, \dfrac{1}{2} = \dfrac{1}{4} = P(A \cap B)$

$\therefore$ A and B are independent events. 1

SECTION – C

12. Let $\sin^{-1}\dfrac{12}{13} = x$, $\cos^{-1}\dfrac{4}{5} = y$ and $\tan^{-1}\dfrac{63}{16} = z$

$\Rightarrow \sin x = \dfrac{12}{13}$, $\cos y = \dfrac{4}{5}$ and $\tan z = \dfrac{63}{16}$ 1

$\therefore \cos x = \sqrt{1 - \sin^2 x} = \dfrac{5}{13}$, $\sin y = \dfrac{3}{5}$

$\therefore \tan x = \dfrac{12}{5}$ and $\tan y = \dfrac{3}{4}$ 1

$$\therefore \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{12}{3} + \frac{3}{4}}{1 - \frac{12}{5} - \frac{3}{4}}$$

$$= -\frac{63}{16} = -\tan z$$

$$= \tan(\pi - 2)$$

$\Rightarrow x + y = \pi - 2 \Rightarrow x + y + z = \pi$ 2

13. Let $y = (\log x)^x + x^{\log x} = u + v$ where $u = (\log x)^x$ and $v = x^{\log x}$

$$\therefore \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$u = (\log x)^x \Rightarrow \log u = x \log(\log x)$$

Diff. w.r.t. x,

$$\frac{1}{u} \cdot \frac{du}{dx} = 1 \cdot \log(\log x) + x \cdot \frac{1}{\log x} \cdot \frac{1}{x}$$ 2

$$\Rightarrow \frac{du}{dx} = u\left[\log(\log x) + \frac{1}{\log x}\right]$$

Similarly for $\frac{dv}{dx}$ 2

14. $f'(x) = \cos x - \sin x = 0 \Rightarrow \tan x = 1 \Rightarrow x = \frac{\pi}{4}, \frac{5\pi}{4}$

$\therefore$ The points $x = \frac{\pi}{4}$ and $x = \frac{5\pi}{4}$ divide the interval $[0, 2\pi]$ into three disjoint intervals $\left[0, \frac{\pi}{4}\right]$, $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, 2\pi\right]$ 2

Now $f'(x) > 0 \Rightarrow x \in \left[0, \frac{\pi}{4}\right]$ and $\left[\frac{5\pi}{4}, 2\pi\right]$

and $f'(x) < 0 \Rightarrow x \in \left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ 1

$\therefore f$ is increasing in $\left[0, \frac{\pi}{4}\right) \cup \left(\frac{5\pi}{4}, 2\pi\right]$

and f is decreasing in $\left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$ 1

15. Let the shooter fire x times. In each trial p = probability of hitting the target $= \frac{3}{4}$ and q = probability of not hitting the target $= \frac{1}{4}$.

$P(X = x) = {}^nc_x \ p^x \ . \ q^{(n-x)} = {}^nc_x \left(\frac{3}{4}\right)^n \left(\frac{1}{4}\right)^{n-x}$ 1

Now given that P(hitting the target at least once) > 0.99

i. e. $P(x \geq 1) > 0.99$ 2

$\therefore 1 - P(x = 0) > 0.99$

$\Rightarrow 1 - {}^nc_0 \left(\frac{3}{4}\right)^0 \left(\frac{1}{4}\right)^{n-0} > 0.99$

$\Rightarrow 1 - \frac{1}{4^n} > 0.99 \Rightarrow \frac{1}{4^n} < 1 - 0.99 = 0.01 \Rightarrow 4^n > \frac{1}{0.01}$

$\Rightarrow 4^n > 100$ Thus the minimum value of n to satisfy the inequality $4^n > 100$ is 4

Thus the shooter must fire 4 times. 1

16. Sum of vectors $= (2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}$

Unit vector of their sum $= \dfrac{(2+\lambda)\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{(2+\lambda)^2 + (6)^2 + (-2)^2}}$

1

Scalar product of $\hat{i} + \hat{j} + \hat{k}$ with this unit vector = 1

$$\therefore \frac{(2+\lambda)\,.\,1 + 6.1 - 2.1}{\sqrt{(2+\lambda)^2 + 36 + 4}} = 1 \Rightarrow (\lambda+6)^2 = \lambda^2 + 4\lambda + 44$$

$\Rightarrow \lambda = 1$ 3

SECTION – D

17. Here $A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$ and $B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$|A| = 1(12 - 5) + 1\ (9 + 10) + 2\ (-3 - 8) = 4 \neq 0$

$\therefore$ So A^{-1} exists 1

For Adj A, $A_{11} = 7$, $A_{12} = -19$, $A_{13} = -11$

$A_{21} = 1$, $A_{22} = -1$, $A_{23} = -1$

$A_{31} = -3$, $A_{32} = 11$, $A_{33} = 7$

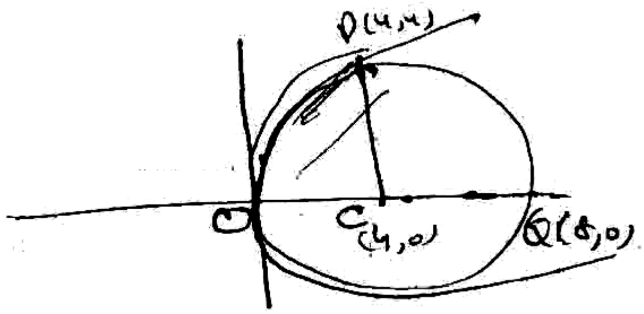
$$\therefore\ adj\,A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$$ 3

$$A^{-1} = \frac{adjA}{|A|}\ \therefore X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}\begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

we get $x = 2$, $y = 1$, $z = 3$ 2

18. $x^2 + y^2 = 8x \Rightarrow x^2 - 8x + y^2 = 0 \Rightarrow (x-4)^2 + y^2 = 16$

$\Rightarrow$ The centre is (4, 0) and radius is 4. Its intersection with the parabola $y^2 = 4x \Rightarrow (x-4)^2 + 4x = 16 \Rightarrow x^2 - 4x = 0 \Rightarrow x = 0$ or $x = 4$. Thus the points of intersection of these two curves are $O(0, 0)$ and $P(4, 4)$ above the x-axis.



The required area of the region *OPQCO* included between these two curves above x-axis is = area of region *OCPO* + area of region *PCQP* 2

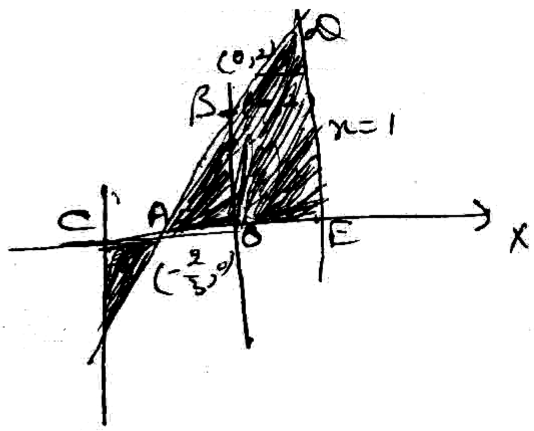
$$= \int_0^4 y\, dx + \int_4^8 9\, dx = 2\int_0^4 \sqrt{x} + \int_4^8 \sqrt{(4)^2 + (x-4)^2}\, dx$$

$$= 2\left(\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right)_0^4 \left[\frac{x-4}{2}\sqrt{4^2 + (x-4)^2} + \frac{16}{2}\sin^{-1}\frac{x-4}{4}\right]_4^8$$

$$= \frac{4}{3}(8 + 3\pi)$$ 4

OR

The line $y = 3x + 2$ meets the x-axis at $x = -\frac{2}{3}$ and y-axis at $y = 2$ and its graph below x-axis for $x \in \left(-1, -\frac{2}{3}\right)$ and above $x = -1$ x-axis for $x \in \left(-\frac{2}{3}, 1\right)$.



2

$\therefore$ The required area $= \left| \int_{-1}^{-\frac{2}{3}} -y\ dx + \int_{-\frac{2}{3}}^{1} y\ dx \right|$

$$= \int_{-1}^{-\frac{2}{3}} (3x+2) + \int_{-\frac{2}{3}}^{-1} 3x + 2dx = \frac{13}{3}$$

4

19. Here $\vec{x_1} = \hat{i} + \hat{j} + \hat{k}$ and $\vec{x_2} = 2\hat{i} + 3\hat{j} + 4\hat{k}$ and $d_1 = 6$, $d_2 = -5$

Using the relation $\vec{r} \cdot \left(\vec{x_1} + \lambda \vec{x_2} \right) = d_1 + \lambda d_2$

we get

$\vec{r}\left[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}\right] = 6 - 5\lambda$ ...... (i) 2

Taking $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$ we get

$\left(x\hat{i} + y\hat{j} + z\hat{k}\right) . \left[(1+2\lambda)\hat{i} + (1+3\lambda)\hat{j} + (1+4\lambda)\hat{k}\right] = 6 - 5\lambda$

$\Rightarrow (1 + 2\lambda)x + (1 + 3\lambda)y + (1 + 4\lambda)z = 6 - 5\lambda$ ..... (ii)

Since (ii) passes through the point (1, 1, 1) So

$(1 + 2\lambda)1 + (1 + 3\lambda) . 1 + (1 + 4\lambda) . 1 = 6 - 5\lambda$

$\Rightarrow 14\lambda = 3 \Rightarrow \lambda = 3/14$ Putting the value of λ in (i)

3

We get $\vec{r}\,.\left[\left(1+\frac{2.3}{14}\right)\hat{i}+\left(1+\frac{3.3}{14}\right)\hat{j}+\left(1+\frac{4.3}{14}\right)\hat{k}\right]$

$\Rightarrow \vec{r}\,.\,(20\hat{i}+23\hat{j}+26\hat{k}) = 69$

$= 6 - 5\,.\,\frac{3}{14}$ 1

OR

Comparing the given lines with $\vec{r} = \vec{a_1} + \lambda\vec{b_1}$ and $\vec{r} = \vec{a_2} + \mu\vec{b_2}$, we get

$\vec{a_1} = \hat{i} + 2\hat{j} + \hat{k}$ and $\vec{b_1} = \hat{i} - \hat{j} + \hat{k}$

$\vec{a_2} = 2\hat{i} - \hat{j} - \hat{k}$ and $\vec{b_2} = 2\hat{i} + \hat{j} + 2\hat{k}$ 2

$\vec{a_2} - \vec{a_1} = -\hat{i} + 3\hat{j} + 2\hat{k}$ and

$$\vec{b_1}\times\vec{b_2} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ 2 & 1 & 2 \end{vmatrix} = -3\hat{i} + 3\hat{k}$$

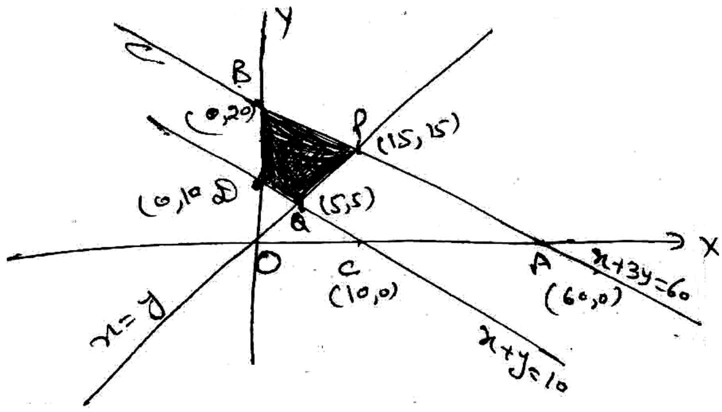
$$\left|\vec{b_1}\times\vec{b_2}\right| = \sqrt{9+9} = 3\sqrt{2}$$

S. D. between the given lines

$$= \left| \frac{\left(\vec{b_1}\times\vec{b_2}\right)\cdot\left(\vec{a_2}-\vec{a_1}\right)}{\left|\vec{b_1}\times\vec{b_2}\right|} \right|$$

$$= \left| \frac{(-1)(-3)-2.3}{3\sqrt{2}} \right| = \frac{9}{3\sqrt{2}} = \frac{3\sqrt{2}}{2} \qquad 4$$

20.



For graph 3

For finding points of intersection *P*, *Q*, *B*, *D* 1

Max. value = 180 at *B*(0, 20) and *P*(15, 15) 1

Min. Value = 60 at *Q*(5, 5) 1

SET – B

SECTION – A

1. (i) **Ans. (B)**

$\because 3x_1 = 3x_2 \Rightarrow x_1 = x_2 \therefore f$ is 1 – 1

and for $f \in R \exists \frac{y}{3}$ in R

s. t. $f\left(\frac{y}{3}\right) = y$. Hence f is onto 1

(ii) **Ans. (A)**

$\because \cos^{-1}\frac{1}{2} + 2\sin^{-1}\frac{1}{2} = \frac{\pi}{3} + 2.\frac{\pi}{6} = \frac{2\pi}{3}$ 1

(iii) **Ans. (C)**

$\because$ $2x - y = 10$ and $3x + y = 5$ solve for x and y

1

$x = 3,\ y = -4$

(iv) **Ans. (B)**

$\because$ A . Adj A = adj A . A = $|A|I \Rightarrow B = \frac{adj\ A}{|A|}$

and $B = A^{-1}$

$\therefore |A^{-1}| = |B| = \left|\frac{adj\ A}{(A)}\right| = \frac{|A|^2}{|A|} = |A|$ 1

(v) **Ans. (B)**

$$\underset{x\to 2^-}{Lt}\ kx^2 = \underset{x\to 2^+}{Lt}\ 3 \Rightarrow 4k = 3 \Rightarrow k = \frac{3}{4}$$ 1

(vi) **Ans. (C)**

Area $= A = \pi r^2 \Rightarrow \frac{dA}{dr} = 2\pi r = 2\pi.4 = 8\pi\text{cm}^2/\text{sec}$

1

(vii) **Ans. (B)**

Diff. w. r. t. x $\frac{2x}{9} + \frac{2y}{16} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx}$

$$= -\frac{2x}{9} \times \frac{16}{2y} = 0$$ 1

$\because$ tangent is x-axis $\therefore \Rightarrow x = 0 \therefore y = \pm 4$

(viii) **Ans. (A)**

$$\int \frac{dx}{x^2+2x+2} = \int \frac{dx}{x^2+2x+1+1} = \int \frac{dx}{(x+1)^2+1}$$

$$= \frac{1}{1}\tan^{-1}\frac{x+1}{1} + c$$ 1

(ix) **Ans. (A)**

$\because x^3 + x\cos x + \tan^5 x$ is odd function

$$\therefore \int_{-\pi/2}^{\pi/2} \left(x^3 + x\cos x + \tan^5 x\right) dx + \int_{-\pi/2}^{\pi/2} 1 \, . \, dx = 0 + \pi$$

1

(x) **Ans. (B)**

$$\frac{dy}{dx} = e^{x+y} = e^x . e^y \Rightarrow e^{-y} . dy = e^x d^x$$

$$\therefore \int e^{-y}\, dy = \int e^x\, dx + c = \frac{e^{-y}}{-1} = e^x + c \qquad 1$$

$$= e^x + e^{-y} + c$$

(xi) **Ans. (D)**

$$y = c_1 e^x + c_2 e^{-x} \Rightarrow \frac{dy}{dx} = c_1 e^x - c_2 e^{-x}$$

$$\frac{d^2y}{dx^2} = c_1 e^x + c_2 e^{-x} = y \Rightarrow \frac{d^2y}{dx^2} - y = 0 \qquad 1$$

(xii) **Ans. (C)**

Projection of $\vec{a} = \hat{i} - \hat{j}$ on $\vec{b} = \hat{i} + \hat{j}$

$$= \frac{|\vec{a} . \vec{b}|}{|\vec{b}|} = \frac{1.1.1.1}{\sqrt{1^2 + 1^2}} = 0 \qquad 1$$

(xiii) **Ans. (B)**

$$\therefore \cos\theta = \frac{1.3 + 2.2 + 2.6}{\sqrt{1^2 + 2^2 + 2^2}\sqrt{3^2 + 2^2 + 6^2}} = \frac{19}{\sqrt{9}\sqrt{49}}$$

$= \frac{19}{3.7} = \frac{19}{21}$

$\therefore \theta = \cos^{-1}\left(\frac{19}{21}\right)$ 1

(xiv) **Ans. (D)**

$\because$ Mean $\frac{1.3+2.2+5.1}{3+2+1} = \frac{12}{16} = 2$ 1

(xv) **Ans. (D)**

$\because P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{P(A \cap B)}{0}$

not defined 1

(xvi) **Ans. (A)**

$\because$ 1, 3, 5 are odd number

$\therefore P(X = 5) = {}^6c_5\left(\frac{1}{2}\right)^5\left(\frac{1}{2}\right)^{6.5}$

$= 6 \cdot \frac{1}{32} \cdot \frac{1}{2} = \frac{3}{32}$ 1

SECTION – B

2. $y = [4, \infty) = \{y \in [4, \infty),\ y = x^2 + 4\} \Rightarrow y = x^2 + 4$

$$\Rightarrow x^2 = y - 4$$

$\therefore\ x = \pm\sqrt{y-4}\ \because\ x \in R_+\ \therefore\ x = \sqrt{y-4} = g(y)$

Let $f(x) = x^2 + 4$ and $g(x) = \sqrt{x-4}$ 1

$\therefore\ fog(x) = f\left(\sqrt{x-4}\right) = \sqrt{(x-4)^2 + 4}\ . = x - 4 + x =$

$$x = I_x$$

$$gof(x) = g\left(x^2 + 4\right) = \sqrt{x^2 + 4 - 4} = x = I_x$$

$\therefore\ g$ is the inverse of f and hence $f'(x) = \sqrt{x-4}$ 1

3. Let $\sin^{-1}\dfrac{8}{17} = x \Rightarrow \sin x = \dfrac{8}{17}$

$$\therefore \cos x = \sqrt{1 - \sin^2 x} = \sqrt{1 - \frac{64}{289}} = \frac{15}{17}$$

and $\sin^{-1}\dfrac{3}{5} = y \Rightarrow \sin y = \dfrac{3}{5}$

$$\therefore \cos y = \sqrt{1 - \sin^2 y} = \sqrt{1 - \frac{9}{25}} = \frac{4}{5}$$

$$\therefore \tan x = \frac{\sin x}{\cos x} = \frac{8}{15} \text{ and } \tan y = \frac{\sin y}{\cos y} = \frac{3}{4}$$ 1

$$\therefore \tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y} = \frac{\frac{8}{15} + \frac{3}{4}}{1 - \frac{8}{15} \cdot \frac{3}{4}} = \frac{32+45}{60-24}$$

$$= \frac{77}{36}$$

$$\therefore x + y = \tan^{-1}\frac{77}{36} \Rightarrow \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$$

1

4. $$A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix} = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$$

$$\therefore (A - A')' = \begin{bmatrix} 0 & -4 \\ 4 & 0 \end{bmatrix} = -\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix} = (A - A')$$

$\therefore$ $A - A'$ is a skew-symmetric matrix 2

5. $$\begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix} = \begin{vmatrix} 1 & 0 & 0 \\ 2 & -1 & 1 \\ 5 & -1 & 1 \end{vmatrix}$$

Operating $C_2 - C_1$ and $C_3 + Cc_1$

$$= 1\begin{vmatrix} -1 & 1 \\ -1 & 1 \end{vmatrix} = -1 + 1 = 0$$ 2

6. Let $I = \int \frac{dx}{\sqrt{6-x-x^2}} = \int \frac{dx}{\sqrt{6-\frac{1}{4}-x-x^2-\frac{1}{4}}}$

$$= \int \frac{dx}{\frac{25}{4}-\left(x+\frac{1}{2}\right)^2} \qquad 1$$

Put $x+\frac{1}{2} = t \Rightarrow dx = dt$ and $a = \frac{5}{2}$

$$\therefore\ I = \int \frac{dt}{\sqrt{a^2-t^2}} = \sin^{-1}\frac{t}{a} + c = \sin^{-1}\frac{\left(x+\frac{1}{2}\right)}{\frac{5}{2}} + c$$

$$= \sin^{-1}\frac{2x+1}{5} + c \qquad 1$$

7. $\int_0^{\pi/2} \frac{dx}{1+\cos x} = \int_0^{\pi/2} \frac{dx}{2+\cos^2\frac{x}{2}}$

$$= \frac{1}{2}\int_0^{\pi/2} \sec^2\frac{x}{2}\,dx = \frac{1}{2}\left[\frac{\tan\frac{x}{2}}{\frac{1}{2}}\right]_0^{\pi/2}$$

$$= \left[\tan\frac{\frac{\pi}{2}}{2} - \tan\frac{0}{2}\right] = \tan\frac{\pi}{4} = 1 \qquad 2$$

8. The general equation of the circles is $x^2 + y^2 + 2gx + 2fy + c = 0$

Since the circle passes through the origin (0, 0)

$\therefore$ It becomes $x^2 + y^2 + 2gx + 2fy = 0 \quad \because c = 0$

Its centre is $(-g, -f)$ but centre lies on y-axis

$\therefore -g = 0 \Rightarrow g = 0$ eq. of the circle becomes

$x^2 + y^2 + 2fy = 0$ Diff. w. r. t. x, we get

$$2x + 2y \cdot \frac{dy}{dx} + 2f \cdot \frac{dy}{dx} = 0 \Rightarrow f = -\frac{x + y \cdot \frac{dy}{dx}}{\frac{dy}{dx}}$$ 1

$\therefore$ The diff. equation becomes

$$x^2 + y^2 + 2\left(-\frac{x + y\frac{dy}{dx}}{\frac{dy}{dx}}\right)y = 0$$

$$\Rightarrow (x^2 + y^2)\frac{dy}{dx} - 2xy - 2y^2 \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow (x^2 - y^2)\frac{dy}{dx} - 2xy = 0$$ 1

9. $y = e^{-x} + ax + b$ Diff. w. r. t. x we get

$\frac{dy}{dx} = -e^{-x} + a$ Again differentiating, we get

$\frac{d^2y}{dx^2} = e^{-x} \Rightarrow e^x \cdot \frac{d^2y}{dx^2} = 1$ Hence the result. 2

10. $\dfrac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$ and $\dfrac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$ 1

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{\cos\theta - 2\cos 2\theta}{-\sin\theta + 2\sin 2\theta}$$ 1

11. Let E be the event that 'number 4 appears at least once' and F be the event that 'the sum of numbers appearing is 6'

Then E = {(4, 1), (4, 2), (4, 3), (4, 4), (4, 5), (4, 6), (1, 4), (2, 4), (3, 4), (5, 4), (6, 4)}

and F = {(1, 5), (2, 4), (3, 3), (4, 2), (5, 1)} 1

$\therefore P(E) = \dfrac{11}{36}$ and $P(F) = \dfrac{5}{36}$ $E \cap F$ = {(2, 4), (4, 2)}

$$\therefore P(E \cap F) = \frac{2}{36}$$

Hence the required probability

$$= P(E/F) = \frac{P(E \cap F)}{P(F)} = \frac{2}{5}$$ 1

SECTION – C

12. L. H. S. $\tan^{-1}\dfrac{1}{5} + \tan^{-1}\dfrac{1}{7} + \tan^{-1}\dfrac{1}{3} + \tan^{-1}\dfrac{1}{8}$

$$= \left(\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)\right) + \tan^{-1}\left(\frac{1}{7}\right) + \tan^{-1}\left(\frac{1}{3}\right)$$

$$\tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{8}}{1-\frac{1}{5}\cdot\frac{1}{8}}\right)+\tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{3}}{1-\frac{1}{7}\cdot\frac{1}{3}}\right)$$ 2

$$= \tan^{-1}\left(\frac{13}{39}\right)+\tan^{-1}\left(\frac{10}{20}\right) = \tan^{-1}\frac{1}{3}+\tan^{-1}\frac{1}{2}$$

$$\tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{2}}{1-\frac{1}{3}\cdot\frac{1}{2}}\right) = \tan^{-1}\frac{5}{5} = \tan^{-1}1 = \frac{\pi}{4}$$ R. H. S. 2

13. Let $y = (\sin x)^x + \sin^{-1}\sqrt{x} = u + v$ where $u = (\sin x)^x$ and $v = \sin^{-1}\sqrt{x}$

$$\frac{dy}{dx} = \frac{du}{dx}+\frac{dv}{dx}$$

Now $u = (\sin x)^x \Rightarrow \log u = x\log(\sin x)$

Diff. w. r. t. x

$$\frac{1}{u}\cdot\frac{du}{dx} = 1\,.\,\log(\sin x) + x\,.\frac{1}{\sin x}\,.\,\cos x = \log(\sin x) + x\cot x$$

$$\Rightarrow \frac{du}{dx} = (\sin x)^x[\log(\sin x) + x\cot x]$$ 2

$$\frac{dv}{dx} = \frac{1}{\sqrt{1-x}} \cdot \frac{1}{2\sqrt{x}}$$

$$\therefore \frac{dy}{dx} = (\sin x)^x [\log(\sin x) + x \cot x] + \frac{1}{2} \cdot \frac{1}{\sqrt{x}\sqrt{1-x}}$$ 2

14. $f'(x) = 6x^2 - 30x + 36 = 6(x^2 - 5x + 6)$

For extreme values

$f'(x) = 0 \Rightarrow x^2 - 5x + 6 = 0 \Rightarrow (x-3)(x-2) = 0$ 1

$\Rightarrow x = 3$ and $x = 2$

Now $f(1) = 2 - 15 + 36 + 1 = 24$

$f(2) = 16 - 60 + 72 + 1 = 29$

$f(3) = 54 - 135 + 108 + 1 = 28$

$f(5) = 250 - 375 + 180 + 1 = 56$ 2

$\therefore$ Absolute maximum value at $x = 5$ is 56

and absolute minimum value at $x = 1$ is = 24 1

15. Let S denote the success (getting a 6) and f denote the failure (not getting a 6). Thus $P(S) = \frac{1}{6}$ and $P(F) = \frac{5}{6}$.

P(A wins in the first throw) $= P(S) = \frac{1}{6}$ 1

A gets the third throw when the first throw by A and 2nd throw by B result into failure.

$\therefore$ P(A wins in 3rd throw) = $P(FFS) = P(F)\ P(F)\ P(S)$

$$= \frac{5}{6} \times \frac{5}{6} \times \frac{1}{6} = \left(\frac{5}{6}\right)^2 \frac{1}{6}$$ 1

P(A wins in 5th throw) = $P(FFFFS)$

$$= P(F)\ P(F)\ P(F)\ P(F)\ P(S)$$

$$= \left(\frac{5}{6}\right)^4 . \frac{1}{6} \text{ and so on}$$

Hence P(A wins) $= \frac{1}{6} + \left(\frac{5}{6}\right)^2 . \frac{1}{6} + \left(\frac{5}{6}\right)^4 . \frac{1}{6} + ..$

$$= \frac{1}{6}\left[1 + \left(\frac{5}{6}\right)^2 + \left(\frac{5}{6}\right)^4 + \ldots\right]$$

$$= \frac{1}{6}\left[\frac{1}{1 - \frac{25}{36}}\right] = \frac{36}{11} \times \frac{1}{6} = \frac{6}{11}$$

$\therefore$ P(B wins) = 1 – P(A wins) $= 1 - \frac{6}{11} = \frac{5}{11}$ 2

16. Vector perpendicular to both $\vec{a}$ and $\vec{b}$ is $\vec{a} \times \vec{b}$

$$\therefore \vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & -1 & 1 \\ 3 & 4 & -1 \end{vmatrix} = \hat{i}(1-4) - \hat{j}(-2-3) + \hat{k}(8+3)$$

$$= -3\hat{i} + 5\hat{j} + 11\hat{k}$$

$$|\vec{a} \times \vec{b}| = \sqrt{(-3)^2 + (5)^2 + (11)^2} = \sqrt{155}$$

$\therefore$ Unit vector perpendicular to $\vec{a}$ and $\vec{b}$ $= \dfrac{\vec{a} \times \vec{b}}{|\vec{a} \times \vec{b}|}$

2

$$= -\frac{3}{\sqrt{155}}\hat{i} + \frac{5}{\sqrt{155}}\hat{j} + \frac{11}{\sqrt{155}}\hat{k}$$

Now $\vec{a} \times \vec{b} = |\vec{a}|\ |\vec{b}| \sin\theta\ \hat{n}$

$\therefore$ $|\vec{a} \times \vec{b}| = |\vec{a}|\ |\vec{b}|$ since

$$\therefore \sin\theta = \frac{|\vec{a} \times \vec{b}|}{|\vec{a}||\vec{b}|} = \frac{\sqrt{155}}{\sqrt{(2)^2 + (-1)^2 + (1)^2}\sqrt{3^2 + 4^2 + (-1)^2}}$$

$$= \frac{\sqrt{155}}{\sqrt{156}} = \sqrt{\frac{155}{156}}$$ 2

SECTION – D

17. The system of equations can be written as $AX = B$

where $A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}$ and $B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$

$|A| = 2(4+1) - 3(-2-3) + 3(-1+6) = 40 \neq 0$

$\therefore\ A^{-1}$ exists 1

Cofactors $A_{11} = 5,\ \ A_{12} = -5,\ \ A_{13} = 5$

$A_{21} = 3,\ \ A_{22} = -13,\ \ A_{23} = +11$

$A_{31} = 9,\ \ A_{32} = 1,\ \ A_{33} = -7$

$$\therefore A^{-1} = \frac{adjA}{|A|} = \frac{1}{40}\begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$ 3

$$\therefore X = A^{-1}B = \frac{1}{30}\begin{bmatrix} 5 & 3 & 9 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25-12+27 \\ 25+52+3 \\ 25-44-21 \end{bmatrix}$$

$$= \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix} \quad \therefore\ x = 1,\ y = 2,\ z = -1$$ 2

18. The given curves are $y = x^2 + 5$ and $y = x^3$ for $1 \le x \le 2,\ y = x^2 + 5 \ge 0$

$\therefore$ graph of the curve $y = x^2 + 5$ lies above x-axis

graph of the curve $y = x^3$ also lies above x-axis for $1 \le x \le 2,\ x^2 + 5 > x^3$ 2

$\therefore$ graph of $x^2 + 5$ lies above the graph of x^3.

$\therefore$ Required area $= \int_1^2 (y_{\text{upper}} - y_{\text{lower}})\, dx$

$$= \int_1^2 (x^2 + 5 - x^3)\, dx = \left.\frac{x^3}{3} + 5x - \frac{x^4}{4}\right|_1^2$$

$$= \frac{8}{3} + 10 - \frac{16}{4} - \frac{1}{3} - 5 + \frac{1}{4} = \frac{43}{12} \text{ sq. units}$$

4

OR

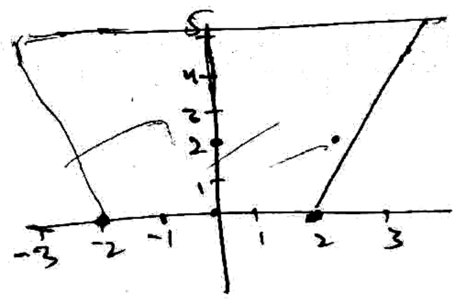
The given curves is $y = x^2 - 4 \Rightarrow x^2 = y + 4$

Since the curve contains only even power of x

Thus it is symmetrical about y-axis. The following table gives some values of x and y satisfying the equation 2

y	0	2	3	4	5
x	± 2	$\pm\sqrt{6}$	$\pm\sqrt{7}$	$\pm\sqrt{8}$	± 3

Plotting these pts and joining them, we get the rough sketch



$\therefore$ Required area $= 2\int_0^5 x\, dy = 2\int_0^5 \sqrt{y+4}\, dy$ 4

$= \dfrac{76}{3}$ sq. units

19. The equation of plane passing through the intersection of given planes is

$$\left[\vec{r}\cdot\left(\hat{i}+3\hat{j}+\hat{k}\right)-5\right]+\lambda\left[\vec{r}\cdot\left(2\hat{i}-\hat{j}+\hat{k}-3\right)\right]=0$$

$\Rightarrow \vec{r}\left[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(-1+\lambda)\hat{k}-5-\lambda\right]=0$ 2

Now since this passes through the point (2, 1, –2)

$\therefore 2\hat{i}+\hat{j}-2\hat{k}$ must satisfy it

$$\therefore \left(2\hat{i}+\hat{j}-2\hat{k}\right).\left[(1+2\lambda)\hat{i}+(3-\lambda)\hat{j}+(-1+\lambda)\hat{k}-5-3\lambda\right]=0$$

$\Rightarrow 2(1 + 2\lambda) + 1(3 - \lambda) - 2(-1 + \lambda) - 5 - 3\lambda = 0$

$\Rightarrow -2\lambda + 2 = 0 \Rightarrow \lambda = 1$ 3

$\therefore$ equation of the required plane is

$\vec{r} = \left[3\hat{i} + 2\hat{j} + 0.\hat{k}\right] = 8$ 1

OR

The given lines are

$\vec{r} = 2\hat{i} + 3\hat{j} + \hat{k} + \lambda\left(2\hat{i} + 5\hat{j} + 3\hat{k}\right)$ .... (i)

and $\vec{r} = 3\hat{i} + 4\hat{j} + 5\hat{k} + \lambda\left(4\hat{i} + 10\hat{j} + 6\hat{k}\right)$ .... (ii)

Now lines (i) and (ii) pass through the points having position vectors $\vec{a_1} = 2\hat{i} + 3\hat{j} + \hat{k}$ and $\vec{a_2} = 3\hat{i} + 4\hat{j} + 5\hat{k}$ respectively and both are parallel to the vector $\vec{b} = 2\hat{i} + 5\hat{j} + 3\hat{k}$ 2

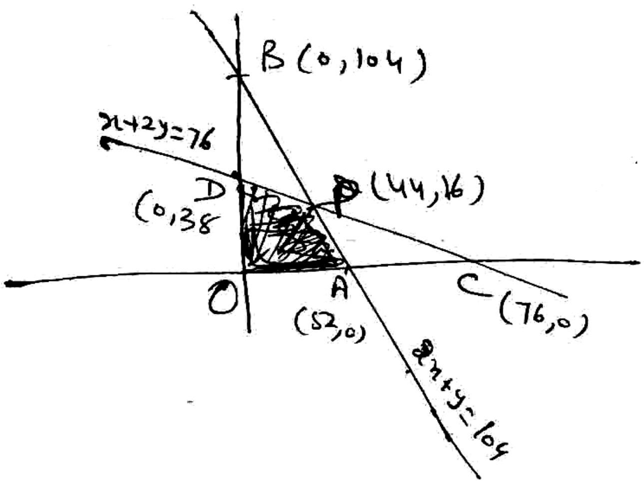
$\therefore \vec{a_2} - \vec{a_1} = \hat{i} + \hat{j} + 4\hat{k}$

Shortest distance S. D. = $\left| \dfrac{\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}}{|\vec{b}|} \right|$

$$= \frac{\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 4 \\ 2 & 5 & 3 \end{vmatrix}}{\sqrt{2^2+5^2+3^2}} = \frac{\left| -17\hat{i}+5\hat{j}+3\hat{k} \right|}{\sqrt{38}}$$

$$= \frac{\sqrt{(-17)^2+(5)^2+(3)^2}}{\sqrt{38}} = \frac{\sqrt{328}}{\sqrt{38}} = \sqrt{\frac{17}{2}}$$ 4

20. 3



3

For finding points of intersection *O*, *A*, *P* and *D* 1

Find value of *Z* at all these points 1

For finding maximum value of *Z* 1

SET – C

SECTION – A

1. (i) **Ans. (B)**

$\because a * b = a^3 + b^3 = b * a = b^3 + a^3$

$\therefore$ * is commutative

and $a * (b * c) = a * \left(b^3 + c^3\right) = a^3 \left(b^3 + c^3\right)^3$

and $(a * b) * c = \left(a^3 + b^3\right)^3 + c^3 \;\; \therefore \; (a * b * c) \neq (a*b) * c$

1

(ii) **Ans. (A)**

$$\because \tan^{-1}\sqrt{3} - \cot^{-1}\left(\sqrt{3}\right) = \tan^{-1}\sqrt{3} - \left(\pi - \cot^{-1}\sqrt{3}\right)$$

$$= \frac{\pi}{3} - \left(\pi - \frac{\pi}{6}\right)$$

$$= \frac{\pi}{3} - \frac{5\pi}{6} = -\frac{\pi}{2}$$ 1

(iii) **Ans. (D)**

$\because \; 2x + 3 = 7 \Rightarrow x = 2$ and $2y - 4 - 14 \Rightarrow 2y = 18$

$\Rightarrow y = 9$ 1

(iv) **Ans. (C)**

$$adjA = \begin{bmatrix} -4 & -3 \\ -1 & 2 \end{bmatrix}$$

$\because A_{11} = -4,\ A_{12} = -1,\ A_{21} = -3,\ A_{22} = 2$ 1

(v) **Ans. (A)**

$$\underset{x \to 5^-}{Lt} f(x) = \underset{x \to 5^+}{Lt} f(x) \ \therefore 5k + 1 = 10 \Rightarrow k = \frac{9}{5}$$

1

(vi) **Ans. (B)**

$$C = 2\pi r \Rightarrow \frac{dC}{dt} = 2\pi \cdot \frac{dr}{dt} = 2\pi(0.7) = 1.4\pi \text{ cm/sec}$$

1

(vii) **Ans. (B)**

$$\frac{2x}{9} + \frac{2y}{16} \cdot \frac{dy}{dx} = 0 \Rightarrow \frac{dy}{dx} = -\frac{2x}{9} \times \frac{16}{2y} = -\frac{16}{9}\frac{x}{y}$$

$\because$ tangent is $\parallel y$-axis $\therefore \dfrac{1}{\frac{dr}{dx}} = 0 \Rightarrow \dfrac{9y}{-16x} = 0$ 1

$\Rightarrow y = 0 \ \therefore \ x = \pm 3$

(viii) **Ans. (D)**

Let $I = \int \dfrac{e^x(1 + x)}{\cos^2(e^x \cdot x)}\, dx$

Put $e^x.x = t \Rightarrow \left(e^x \cdot x + 1.e^x\right) dx = dt$

$$\Rightarrow e^x(1 + x)\, dx = dt$$

$$\therefore\ I = \int \frac{dt}{\cos^2 t} = \int \sec^2 t = \tan t + c$$

$$= \tan\left(e^x \,.\, x\right) + c \qquad 1$$

(ix) **Ans. (A)**

$$\because \int_0^{2/3} \frac{dx}{4+9x^2} = \frac{1}{9}\int \frac{dx}{\frac{4}{9}+x^2} = \frac{1}{9} \,.\, \frac{3}{2} \tan^{-1} \frac{3x}{2}\Bigg|_0^{2/3}$$

$$= \frac{1}{6}\left[\tan^{-1} 1 - \tan^{-1} 0\right]$$

$$= \frac{1}{6} \,.\, \frac{\pi}{4} = \frac{\pi}{24} \qquad 1$$

(x) **Ans. (A)**

$$y + c\sin x = 0 \Rightarrow \frac{dy}{dx} = -c\cos x \Rightarrow c = -\frac{dy}{dx} \ \cos x$$

$$\therefore y + \left(-\frac{dy}{dx} \,.\, \sec x\right)\sin x = 0$$

$$\Rightarrow y - \tan x \,.\, \frac{dy}{dx} = 0 \qquad 1$$

$$\Rightarrow \frac{dy}{dx} - y\cot x = 0 \Rightarrow \frac{y}{\tan x} - \frac{dy}{dx} = 0$$

(xi) **Ans. (D)**

$$x^2.\frac{dy}{dx} = 2 \Rightarrow dy = \frac{2}{x^2}\,.\,dx \Rightarrow y = \frac{2x^{-2+1}}{-2+1} + c$$

$$= -\frac{2}{x} + c \qquad 1$$

(xii) **Ans. (C)**

$\because$ Vectors are orthogonal

$\therefore$ Scalar product is zero

$\therefore$ $5\lambda + 2\,(-1) + (-1)\,5 = 0 \Rightarrow \lambda = \frac{7}{5}$ 1

(xiii) **Ans. (B)**

$$\therefore \cos\theta = \frac{2.4 + 2.1 + 1.8}{\sqrt{2^2 + 2^2 + 1^2}\sqrt{4^2 + 1^2 + 8^2}} = \frac{18}{\sqrt{9}\sqrt{81}}$$

$$= \frac{18}{3} \qquad 1$$

$$\therefore \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

(xiv) **Ans. (D)**

$\because$ $P(B / A) = 1 \Rightarrow \frac{P(A \cap B)}{P(A)} = 1$ 1

Which is possible of $P(A \cap B) = P(A)$ $\therefore$ $A \subset B$

(xv) **Ans. (C)**

$\because$ a pair of dice is rolled

$\therefore$ No. of outcomes = 36

But prime odd nos. are 3, 5

$\therefore\ P(E) = \frac{4}{36} = \frac{1}{9}$ 1

(xvi) **Ans. (D)**

$\because P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{P(A \cap B)}{0}$

which is not defined 1

SECTION – B

2. $f(x_1) = f(x_2) \Rightarrow \frac{x_1}{x_1+2} = \frac{x_2}{x_2+2} \Rightarrow x_1(x_2+2)$

$= x_2(x_1+2)$

$\Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2$ 1

$\therefore f$ is one-one. Let $f(x) = y \Rightarrow y = \frac{x}{x+2}$

$\Rightarrow y(x+2) = x \Rightarrow yx - x = -2y \Rightarrow x = -\frac{-2y}{y-1}$

$\therefore\ f^{-1}(y) = x = -\frac{2y}{y-1} \quad \therefore f^{-1}(x) = -\frac{2x}{x-1} = \frac{2x}{1-x}$ 1

3. L. H. S.

$$2\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{7}=\tan^{-1}\left(\frac{2\cdot\frac{1}{2}}{1\frac{1}{4}}\right)+\tan^{-1}\frac{1}{7}$$

$$=\tan^{-1}\left(+\frac{4}{3}\right)+\tan^{-1}\frac{1}{7} \qquad 1$$

$$=\tan^{-1}\frac{1}{7}+\tan^{-1}\frac{4}{3}$$

$$=\tan^{-1}\left(\frac{\frac{1}{7}+\frac{4}{3}}{1-\frac{1}{7}\cdot\frac{4}{3}}\right)$$

$$=\tan^{-1}\left(\frac{3+28}{21-4}\right)=\tan^{-1}\frac{31}{17}=\text{ R. H. S.} \qquad 1$$

4. Let $A=\begin{bmatrix}2 & -6\\ 1 & -2\end{bmatrix}\therefore A=IA\Rightarrow\begin{bmatrix}2 & -6\\ 1 & -2\end{bmatrix}=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}A$

$\Rightarrow\begin{bmatrix}1 & -2\\ 2 & -6\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & 0\end{bmatrix}A$ (Operating $R_2\leftrightarrow R_1$)

$\Rightarrow\begin{bmatrix}1 & -2\\ 0 & -2\end{bmatrix}=\begin{bmatrix}0 & 1\\ 1 & -2\end{bmatrix}A$ (Operating R_2-2R_1) 1

$$\Rightarrow \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{1}{2} & 1 \end{bmatrix} A \qquad \left(\text{Operating } -\frac{1}{2} R_2\right)$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix} A \quad \therefore A^{-1} = \begin{bmatrix} -1 & 3 \\ -\frac{1}{2} & 1 \end{bmatrix}$$ 1

5. $\because$ A and B are symmetric $\therefore A' = A$ and $B' = B$

$(AB - BA)' = (AB)' - (BA)' = B'A' - A'B' = BA - AB$

$= -(AB - BA)$

$\therefore$ $AB - BA$ is a skew symmetric matrix 2

6. $\because 8 + 2x - x^2 = 8 + 1 - 1 + 2x - x^2 = 9 - (x-1)^2$

$\int \sqrt{8 + 2x - x^2}\ dx = \int \sqrt{9 - (x-1)^2} = \int \sqrt{9 - t^2}\ dt$

Put $x - 1 = t \Rightarrow dx = dt$ 1

$\therefore \int \sqrt{9 - t^2}\ dt = \frac{t}{2}\sqrt{3^2 - t^2} + \frac{9}{2}\sin^{-1}\frac{t}{3} + c$

$\Rightarrow I = \frac{x-1}{2}\sqrt{9 - x^2 + 2x - 1} + \frac{9}{2}\sin^{-1}\frac{x-1}{3} + c$ 1

$= \frac{x-1}{2}\sqrt{8 - x^2 + 2x} + \frac{9}{2}\sin^{-1}\frac{x-1}{3} + c$

7. Let $I = \int_0^{\pi/4} \tan^3 x\, dx = \int_0^{\pi/4} \tan x \,.\, \tan^2 x\, dx$

$$= \int_0^{\pi/4} \tan x \left(\sec^2 x - 1\right) dx \qquad 1$$

$$= \int_0^{\pi/4} \tan x \,.\, \sec^2 x\, dx - \int_0^{\pi/4} \tan x\, dx = \frac{\tan^2 x}{x}$$

$$\Big|_0^{\pi/4} \Big| - \log \sec x \Big|_0^{\pi/4}$$

$$= \frac{1}{2} - \log\sqrt{2} = \frac{1}{2} - \frac{1}{2}\log 2$$

$$= \frac{1}{2}(1 - \log 2) \qquad 1$$

8. The general equation of circles is $x^2 + y^2 + 2gx + 2fy + c \Rightarrow 0$

Since it passes through the origin

$\therefore$ $c = 0$ and centre is on x-axis. Hence centre $(-g, -f) \Rightarrow -f = 0 \Rightarrow f = 0$ 1

$\therefore$ equation becomes $x^2 + y^2 + 2gx = 0$

Diff. w. r. t. x, we get $2x + 2y\frac{dy}{dx} + 2g = 0$

$\therefore$ $g = -\left(x + y \,.\, \frac{dy}{dx}\right) = 0$

$\therefore$ The diff. equation is

$$x^2 + y^2 + 2x\left(-x - y \,.\, \frac{dy}{dx} = 0\right)$$

$$\Rightarrow x^2 - y^2 + 2xy\frac{dy}{dx} = 0$$ 1

9. The diff. equation is $ydx = xdy - xydx$

Dividing by xy, we get $\frac{dx}{x} - \frac{dy}{y} = dx$ 1

Integrating both sides, $\log x - \log y = x + c$

$$\Rightarrow \log\left(\frac{x}{y}\right) = x + c$$ 1

10. $f(x) = x^2 + 2x - 8$, $x \in [-4, -2]$ is a polynomial

$\therefore f(x)$ is continuous in $[-4, -2]$ and derivable in $(-4, 2)$ and $f(-4) = 16 - 8 - 8 = 0$ and $f(2) = 4 + 4 - 8 = 0$

$\Rightarrow f(-4) = f(2)$ $\therefore$ Rolle's theorem is satisfied 1

$\therefore$ a point $c \in (-4, 2)$ s. t. $f'(c) = 0 \Rightarrow 2c + 2 = 0$

$\Rightarrow c = -1 \in (-4, 2)$ 1

11. Let A be the event 'the number on the card drawn is even' and B be the event 'the number on the card drawn > 3'

We have to find P(A/B)

The sample space of experiment is = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

A = {2, 4, 6, 8, 10}; B = {4, 5, 6, 7, 8, 9, 10}

$A \cap B = \{4, 6, 8, 10\}$ 1

$\therefore P(A) = \frac{5}{10} = \frac{1}{2}$ and $P(B) = \frac{7}{10}$, $P(A \cap B) = \frac{4}{10}$

$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{4}{10} \times \frac{10}{7} = \frac{4}{7}$ 1

SECTION – C

12. Let $\sin^{-1}\frac{1}{5} = z$ and $\cos^{-1} x = y$

$\Rightarrow \sin z = \frac{1}{5}$ and $\cos y = x$

$\cos z = \sqrt{1 - \sin^2 z}$ $\sin y = \sqrt{1 - \cos^2 y} = \sqrt{1 - x^2}$ 1

$= \sqrt{1 - \left(\frac{1}{5}\right)^2} = \frac{\sqrt{24}}{5}$

$\therefore$ The given equation becomes

$\sin(z + y) = 1 \Rightarrow \sin z \cdot \cos y + \cos z \sin y = 1$

$\Rightarrow \frac{1}{5} \cdot x + \frac{\sqrt{24}}{5}\sqrt{1 - x^2} = 1$

$$\Rightarrow \frac{\sqrt{24}}{5}\sqrt{1-x^2} = 1-\frac{x}{5} \text{ squaring}$$

$$\Rightarrow \frac{24}{25}\left(1-x^2\right) = \left(1-\frac{x}{5}\right)^2 \Rightarrow \frac{24}{25}-\frac{24}{25}x^2 = \frac{x^2}{25}-\frac{2x}{5}+1$$

$$\Rightarrow 25x^2-10x+1=0 \Rightarrow (5x-1)^2=0 \Rightarrow x=\frac{1}{5}$$ 3

13. $y^x + x^x + x^y = a^b \Rightarrow u+v+w = a^b$ where

$u = y^x, v = x^x$ and $w = x^y$ and $\frac{du}{dx}+\frac{dv}{dx}+\frac{dw}{dx}=0$

$\log u = x \log y$

$\Rightarrow \frac{1}{u}\frac{dy}{dx} = \log y + x \,.\, \frac{1}{y}\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = y^x\left(\log y + \frac{x}{y}\frac{dy}{dx}\right)$

$v = x^x$

$\Rightarrow \log v = x\log x$

$\frac{1}{v}\frac{dv}{dx} = \log x + x \,.\, \frac{1}{x}$

$= 1 + \log x$

$\Rightarrow \frac{dv}{dx} = x^x(1+\log x)$

$w = x^y \Rightarrow \log w = y\log x$

Diff. w. r. t. $x \Rightarrow$

$\frac{1}{w}\frac{dw}{dx} = \frac{y}{x} + \log x \,.\, \frac{dy}{dx} \Rightarrow \frac{dw}{dx} = x^y\left[\frac{y}{x}+\log x\frac{dy}{dx}\right]$

2

$$\frac{du}{dx}+\frac{dv}{dx}+\frac{dw}{dx}=0$$

$$\Rightarrow y^x\left[\log x+\frac{x}{y}\frac{dy}{dx}\right]+x^x(1+\log x)+$$

$$x^y\left[\frac{y}{x}+\log x.\frac{dy}{dx}\right]=0$$

$$\Rightarrow\left[y^x.\frac{x}{y}+x^y\log x\right]\frac{dy}{dx}-\left(y^x\log y+x^x(1+\log x)-y^y.\frac{y}{x}\right)$$

$$\Rightarrow\frac{dy}{dx}=-\frac{y^x\log y+x^x(\log x)+x^{y-1}\ .\ y}{xy^{x-1}+x^y\ .\log x}$$ 2

14. Here

$f(x)=2x^3-3x^2-36x+7\Rightarrow f'(x)=6x^2-6x-36$

$\Rightarrow f'(x) = 6\ (x^2 - x - 6) = 6(x-2)\ (x-3) = 0$

$\Rightarrow x = -2$ and 3

Thus $x = -2$ and 3 divides the real line into three disjoint intervals $(-\infty, -2)$ $(-2, 3)$ and $(3, \infty)$

2

$f'(x) > 0$ in $(-\infty, -2)$ and $(3, \infty)$

$\therefore f$ is strictly increasing in $(-\infty, -2) \cup (3, \infty)$ 1

and $f'(x) < 0$ for $x \in (-2, 3)$

$\therefore f$ is strictly decreasing in $(2, 3)$ 1

15. Let E be the event that the man reports that sin occurs in the throwing of the die and S_1 be the event that sin occurs and S_2 be the event that sin does not occur.

Thus $P(S_1) = \frac{1}{6}$, $P(S_2) = \frac{5}{6}$ and 1

$P(E/S_1)$ = Probability that the man reports that sin occurs when six has actually occurred on the die.

= Probability that the man speaks the truth $= \frac{3}{4}$

$P(E/S_2)$ = Probability that the man reports that sin occurs when six has not actually occurred on the die.

= Probability that the man does not speaks the truth $= 1 - \frac{3}{4} = \frac{1}{4}$ 2

Then by Bayer's theorem, we get

$P(S_1/E)$ = Probability that the report of the man that six has occurred is actually a six

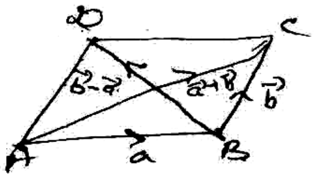
$$= \frac{P(S_1) \, . \, P(E/S_1)}{P(S_1)\, P(E/S_1) + P(S_2)\, P(E/S_2)} = \frac{3}{8}$$ 1

16. Let ABCD be a || gm where $\overrightarrow{AB} = \vec{a}$ and $\overrightarrow{BC} = \vec{b}$

$\therefore\ \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{AC} \Rightarrow \overrightarrow{AC} = \vec{a} + \vec{b}$

$\therefore\ \overrightarrow{AC} = \vec{a} + \vec{b} = (\hat{i} + 2\hat{j} + 3\hat{k}) + (2\hat{i} + 4\hat{j} - 5\hat{k})$

$= 3\hat{i} + 6\hat{j} - 2\hat{k}$



Also $\overrightarrow{AB} + \overrightarrow{BD} = \overrightarrow{AD}$

$\Rightarrow \overrightarrow{BD} = \overrightarrow{AD} - \overrightarrow{AB} = \vec{b} - \vec{a}$

$= (2\hat{i} + 4\hat{j} - 5\hat{k}) - (\hat{i} + 2\hat{j} + 3\hat{k}) = \hat{i} + 2\hat{j} - 8\hat{k}$ 2

Unit Vector along $\overrightarrow{AC} = \dfrac{\overrightarrow{AC}}{|\overrightarrow{AC}|} = \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{\sqrt{3^2 + 6^2 + (-2)^2}}$

$= \dfrac{3\hat{i} + 6\hat{j} - 2\hat{k}}{7}$ 1

Unit Vector along

$\overrightarrow{BD} = \dfrac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{1^2 + 2^2 + (-8)^2}} = \dfrac{\hat{i} + 2\hat{j} - 8\hat{k}}{\sqrt{69}}$ 1

SECTION – D

17. System of linear equations can be written as $AX = B$

where $A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}$ and $B = \begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix}$

$|A| = 2(10 + 3) - 1\,(-5 + 0) + 1\,(3 + 0) = 34 \neq 0$

$\therefore A^{-1}$ exists

To find A^{-1}, find adj A 1

$\therefore\ A_{11} = 13,\quad A_{12} = 5,\quad A_{13} = 3$

$A_{21} = 8,\quad A_{22} = -10,\quad A_{23} = -6$

$A_{31} = 1,\quad A_{32} = 3,\quad A_{33} = -5$

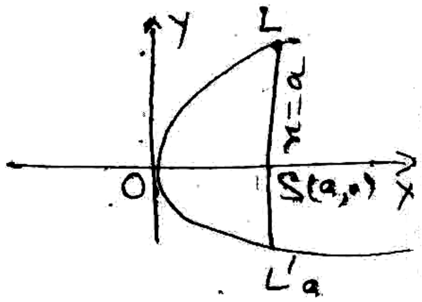
$$\therefore A^{-1} = \frac{adjA}{|A|} = \frac{1}{|A|}\begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

3

$$\therefore X = A^1B = \frac{1}{34}\begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}\begin{bmatrix} 1 \\ \frac{3}{2} \\ 9 \end{bmatrix} = \frac{1}{34}\begin{bmatrix} 13+12+9 \\ 5-15-27 \\ 3-9-45 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{x} \end{bmatrix} \quad \therefore\ x = 1,\ y = \frac{1}{2},\ z = -\frac{3}{2}$$ 2

18. Let S (a, 0) be the focus of the parabola $y^2 = 4ax$. Then LSL' is its latus rectum which is the ling through focus and $\perp$ to x-axis.



Thus the equation of latus rectum is $x = a$

Now the equation of the parabola contains even powers of y.

$\therefore$ It is symmetrical about x-axis 2

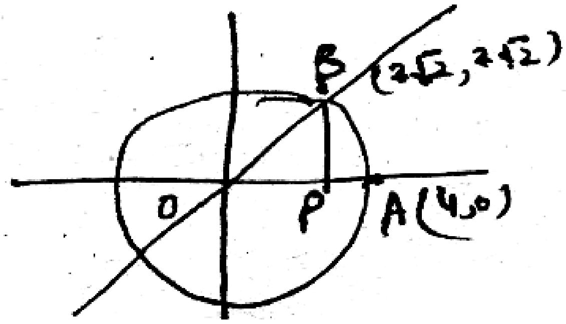
$\therefore$ Required area = Area LOL' = 2 area LOS $= 2\int_0^a y\,dx$

$$= 2\int_0^a 2\sqrt{a}\sqrt{x}\,dx = 4\sqrt{a}\left.\frac{x^{\frac{3}{2}}}{3/2}\right|_0^a = \frac{8}{3}a^2 \text{ sq. units}$$ 4

OR

The equation of given curves are $x^2 + y^2 = 16$ and $y = x$

$x^2 + y^2 = 16$ is a circle with centre (0, 0) and radius 4 and y = represents a st. line passing through the original marks an angle of 45° with x-axis. To find their points of intersection.



$x^2 + y^2 = 16 \Rightarrow x^2 = 8 \Rightarrow x = \pm 2\sqrt{2}$ $\therefore$ $x = 2\sqrt{2}$ in the first quadrant $\therefore$ $y = 2\sqrt{2}$ 2

$\therefore$ Required area = Area OBP + Area $BPAB$

$$= \int_0^{2\sqrt{2}} y \text{ of line } y = x \, dx + \int_{2\sqrt{2}}^{4} y \text{ of curve } dx =$$

$$\int_0^{2\sqrt{2}} x \, dx + \int_{2\sqrt{2}}^{4} \sqrt{(4)^2 - x^2 dx}$$

$$= \frac{x^2}{2}\Big|_0^{2\sqrt{2}} + \frac{x}{2}\sqrt{16-x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\Big|_{2\sqrt{2}}^{4}$$

$$= \left(4 + 8\sin^{-1}1 - \frac{2\sqrt{2}}{4}\sqrt{8} - 8\sin^{-1}\frac{2\sqrt{2}}{4}\right)$$

$$= 4 + 8 \cdot \frac{\pi}{2} - 4 - 8 \cdot \frac{\pi}{4} = 2\pi \text{ sq. units}$$ 4

19. The equation of the planes are

$$\vec{r} \cdot (2\hat{i} - 7\hat{j} + 4\hat{k}) = 3 \quad \ldots\ldots\ldots\ldots \quad \text{(i)}$$

and $\vec{r} \cdot (3\hat{i} - 5\hat{j} + 4\hat{k}) = -11 \quad \ldots\ldots\ldots\ldots \quad \text{(ii)}$

Equation of the plane through the line of intersection of (i) and (ii) is

$$\vec{r} \cdot (3\hat{i} - 5\hat{j} + 4\hat{k}) + \lambda(\vec{r} \cdot 3\hat{i} - 5\hat{j} + 4\hat{k}) = 3 - || \ \lambda$$

$$\vec{r} \cdot ((3 + 3\lambda)\hat{i} + (-5 - 5\lambda)\hat{j} + (4 + 4\lambda)\hat{k}) = 3 - 11\lambda \quad \text{(iii)}$$
2

Let $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$\Rightarrow (3 + 3\lambda)\, x + (-5 - 5\lambda)\ \text{y} + (4 + 4\lambda)z = 3 - ||\ \lambda$

$\Rightarrow -6\lambda - 5\lambda + 12\lambda + 11\lambda = 3 - 12 + 5 + 6 \Rightarrow 12\lambda = 2$ 3

$\Rightarrow \lambda = \frac{1}{6}$ Put in (iii) that will be the required equation of the plane 1

OR

The given lines are

$$\vec{r} = \hat{i} + 2\hat{j} - 4\hat{k} + \lambda(2\hat{i} + 3\hat{j} + 6\hat{k})$$

and $\vec{r} = 3\hat{i} + 3\hat{j} - 5\hat{k} + \mu(2\hat{i} + 3\hat{j} + 6\hat{k})$

Now lines pass through the points having position vectors $\vec{a_1} = \hat{i} + 2\hat{j} - 4\hat{k}$ and $\vec{a_2} = 3\hat{i} + 3\hat{j} - 5\hat{k}$ respectively and both are parallel to the vector $\vec{b} = 2\hat{i} + 3\hat{j} + 6\hat{k}$ 2

$$\text{S. D.} = \left| \frac{\left(\vec{a_2} - \vec{a_1}\right) \times \vec{b}}{|\vec{b}|} \right| \quad \therefore \vec{a_2} - \vec{a_1} = 2\hat{i} + \hat{j} - \hat{k}$$

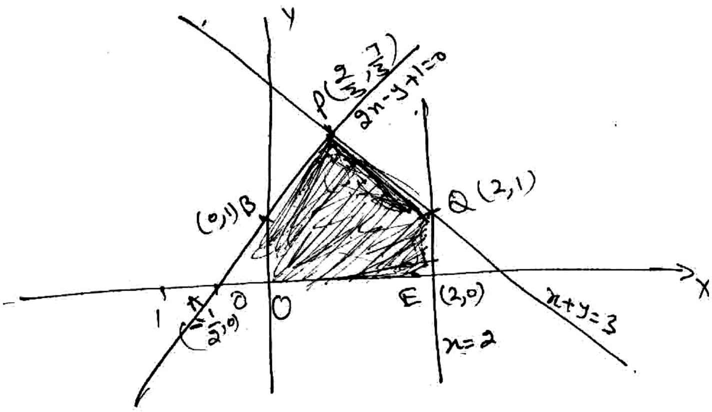
$$\therefore \left(\vec{a_2} - \vec{a_1}\right) \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 2 & 1 & -1 \\ 2 & 3 & 6 \end{vmatrix} = 9\hat{i} - 14\hat{j} + 4\hat{k}$$

$$\therefore \left|\left(\vec{a}_2 - \vec{a}_1\right) \times \vec{b}\right| = \sqrt{9^2 + 14^2 + 4^2} = \sqrt{81 + 196 + 16} = \sqrt{293}$$

$$|\vec{b}| = \sqrt{2^2 + 3^2 + 6^2} = \sqrt{4 + 9 + 36} = 7$$

$$\therefore \text{ S. D. } = \frac{\sqrt{293}}{7}$$ 4

20.



3

The co-ordinates of the extreme points of the feasible region are $O(0, 0)$; $E(2, 0)$; $Q(2, 1)$; $P\left(\frac{2}{3}, \frac{7}{3}\right)$ and $B(0, 1)$ 2

Find Z at all these points

$\therefore$ Z is maximum at Q(2, 1) and $P\left(\frac{2}{3}, \frac{7}{3}\right)$ and maximum value of $Z = 3$ 1