CLASS : 12th (Sr. Secondary) **3681/3631**

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MARKING INSTRUCTIONS AND MODEL ANSWERS

MATHEMATICS

ACADEMIC/OPEN

(Only for Fresh/Re-appear Candidates)

उप-परीक्षक मूल्यांकन निर्देशों का ध्यानपूर्वक अवलोकन करके उत्तर-पुस्तिकाओं का मूल्यांकन करें। यदि परीक्षार्थी ने प्रश्न पूर्ण व सही हल किया है तो उसके पूर्ण अंक दें।

General Instructions :

(i) *Examiners are advised to go through the general as well as specific instructions before taking up evaluation of the answer-books.*

(ii) *Instructions given in the marking scheme are to be followed strictly so that there may be uniformity in evaluation.*

(iii) *Mistakes in the answers are to be underlined or encircled.*

(iv) Examiners need not hesitate in awarding full marks to the examinee if the answer/s is/are absolutely correct.

(v) Examiners are requested to ensure that every answer is seriously and honestly gone through before it is awarded mark/s. It will ensure the authenticity as their evaluation and enhance the reputation of the Institution.

(vi) A question having parts is to be evaluated and awarded partwise.

(vii) If an examinee writes an acceptable answer which is not given in the marking scheme, he or she may be awarded marks only after consultation with the head-examiner.

(viii) If an examinee attempts an extra question, that answer deserving higher award should be retained and the other scored out.

(ix) Word limit wherever prescribed, if violated up to 10%. On both sides, may be ignored. If the violation exceeds 10%, 1 mark may be deducted.

(x) *Head-examiners will approve the standard of marking of the examiners under them only after ensuring the non-violation of the instructions given in the marking scheme.*

(xi) *Head-examiners and examiners are once again requested and advised to ensure the authenticity of their evaluation by going through the answers seriously, sincerely and honestly. The advice, if not headed to, will bring a bad name to them and the Institution.*

महत्त्वपूर्ण निर्देश :

(i) *अंक-योजना का उद्देश्य मूल्यांकन को अधिकाधिक वस्तुनिष्ठ बनाना है। अंक-योजना में दिए गए उत्तर-बिन्दु अंतिम नहीं हैं। ये सुझावात्मक एवं सांकेतिक हैं। यदि परीक्षार्थी ने इनसे भिन्न, किन्तु उपयुक्त उत्तर दिए हैं, तो उसे उपयुक्त अंक दिए जाएँ।*

(ii) *शुद्ध, सार्थक एवं सटीक उत्तरों को यथायोग्य अधिमान दिए जाएँ।*

(iii) परीक्षार्थी द्वारा अपेक्षा के अनुरूप सही उत्तर लिखने पर उसे पूर्णांक दिए जाएँ।

(iv) वर्तनीगत अशुद्धियों एवं विषयांतर की स्थिति में अधिक अंक देकर प्रोत्साहित न करें।

(v) भाषा-क्षमता एवं अभिव्यक्ति-कौशल पर ध्यान दिया जाए।

(vi) मुख्य-परीक्षकों/उप-परीक्षकों को उत्तर-पुस्तिकाओं का मूल्यांकन करने के लिए केवल Marking Instructions/ Guidelines दी जा रही है यदि मूल्यांकन निर्देश में किसी प्रकार की त्रुटि हो, प्रश्न का उत्तर स्पष्ट न हो, मूल्यांकन निर्देश में दिए गए उत्तर से अलग कोई और भी उत्तर सही हो तो परीक्षक, मुख्य-परीक्षक से विचार-विमर्श करके उस प्रश्न का मूल्यांकन अपने विवेक अनुसार करें।

SET – A

SECTION – A

1. (i) $(gof)\ (x) = gof\ (x)$

$= g(\log(1 + x))$

$= e^{\log(1+x)} = 1 + x$ **Ans. (B)** 1

(ii) Let $\cos^{-1}\frac{3}{5} = \theta \Rightarrow \cos\theta = \frac{3}{5}$

$$\sin\left(\cos^{-1}\frac{3}{5}\right) = \sin\theta = \sqrt{1-\cos^2\theta}$$

$$= \sqrt{1-\frac{9}{25}} = \sqrt{\frac{16}{25}} = \frac{4}{5}$$

Ans. (A) 1

(iii) $2A + B = 2\begin{bmatrix} 4 & 2 & 3 \\ 1 & 5 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 7 \\ 0 & 4 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 8 & 4 & 6 \\ 2 & 10 & 14 \end{bmatrix} + \begin{bmatrix} 1 & 3 & 7 \\ 0 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 9 & 7 & 13 \\ 2 & 14 & 15 \end{bmatrix}$$

Ans. (A) 1

(iv) $\begin{vmatrix} 3 & -4 \\ m & 5 \end{vmatrix} = 3 \Rightarrow 15 + 4m = 3$

$$\Rightarrow 4m = -12$$

$\Rightarrow m = -3$ **Ans. (C)** 1

(v) $\frac{d}{dx}\left(\sin x^3\right) = \cos x^3 \, . \, 3x^2$

$= 3x^2 \, . \, \cos x^3$ **Ans. (B)** 1

(vi) $f(x) = \sin 3x + 4$

$3 \le f(x) \le 5$

$\because\ -1 \le \sin 3x \le 1$ **Ans. (A)** 1

(vii) $x = a\cos^3\theta,\ \ y = a\sin^3\theta$

$$\frac{dx}{d\theta} = -3a\cos^2\theta \,.\sin\theta$$

$$\frac{dy}{d\theta} = 3a\sin^2\theta \,.\cos\theta$$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{3a\sin^2\theta\,.\cos\theta}{-3a\cos^2\theta\,.\sin\theta} = -\tan\theta$$

at $\theta = \frac{\pi}{4}, \frac{dy}{dx} = -1$ **Ans. (C)** 1

(viii) $\int \tan^2 x\, dx = \int \{\sec^2 x - 1\}\, dx$

$= \tan x - x + c$ **Ans. (A)** 1

(ix) $I = \int \frac{3x}{1+2x^4}\, dx$

Put $x^2 = t, \quad 2x\, dx = dt$

$$\therefore I = \frac{3}{2}\int \frac{dt}{1+2t^2} = \frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}\ t\right)+c$$

$$= \frac{3}{2\sqrt{2}}\tan^{-1}\left(\sqrt{2}x^2\right)+c$$

Ans. (B) 1

(x) Degree 1 **Ans. (C)** 1

(xi) $\frac{dy}{dx} = \tan^2 x$

Integrate w. r. t. x, we get

$y = \int \tan^2 x\ dx + c$

$= \int \left(\sec^2 x - 1\right) dx + c$

$\Rightarrow y = \tan x - x + c$ **Ans. (A)** 1

(xii) $P(A/B) = \frac{P(A\cap B)}{P(B)} = \frac{\frac{4}{13}}{\frac{9}{13}} = \frac{4}{9}$ **Ans. (C)** 1

(xiii) Required Prob. $= \frac{4}{52}\times\frac{3}{51}$

$= \frac{1}{13}\times\frac{1}{17} = \frac{1}{221}$ **Ans. (B)** 1

(xiv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \,.\, P(B)$

$\Rightarrow 0.60 = 0.2 + P(B)[1 - 0.2]$

$\Rightarrow P(B) = \frac{0.40}{0.80} = \frac{1}{2} = 0.5$ **Ans. (A)** 1

(xv) $\cos\theta = \frac{\vec{a} \,.\, \vec{b}}{|\vec{a}|\,.\,|\vec{b}|} = \frac{1.3 + 2.(-1) + (-3).2}{\sqrt{1+4+9}\sqrt{9+1+4}}$

$= \frac{3-2-6}{\sqrt{14}\sqrt{14}} = \frac{-5}{14}$

$\therefore\ \theta = \cos^{-1}\left(\frac{-5}{14}\right)$ **Ans. (C)** 1

(xvi) $\cos\theta = \frac{4.1 + 3.(-2) + 2.1}{\sqrt{16+9+4}\sqrt{1+4+1}}$

$= \frac{4-6-2}{\sqrt{29}\sqrt{6}} = 0$

$\therefore\ \theta = 90°$ **Ans. (A)** 1

SECTION – B

2. $f(1) = 1, f(2) = 1$

Also $f(-1) = -1, f(-2) = -1$ 1

$\therefore f$ is not one-one 1

3. Let $\sin^{-1} x = \theta$

$\Rightarrow x = \sin\theta$

$= \cos\left(\frac{\pi}{2} - \theta\right)$ 1

$\therefore \cos^{-1} x = \frac{\pi}{2} - \theta$

Now $\sin^{-1} x + \cos^{-1} x = \theta + \frac{\pi}{2} - \theta = \frac{\pi}{2}$. 1

4. $A^2 = A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 9-1 & 3+2 \\ -3-2 & -1+4 \end{bmatrix}$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$ 1

$f(A) = A^2 - 5A + 7I$

$= \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 8-15+7 & 5-5 \\ -5+5 & 3-10+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ 1

5. Area $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} 0 & 0 & 1 \\ -2 & 3 & 1 \\ 10 & 7 & 1 \end{vmatrix} \qquad 1$$

$$= \frac{1}{2}[1(-14-30)] = -\frac{44}{2} = -22$$

$\therefore$ Required area = 22 sq. units. 1

6. Let $y = (\tan x)^{\cot x}$

taking log both side, we get

$\log y = \cot x \cdot \log(\tan x)$ 1

diff. w. r. t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \cot x \cdot \frac{1}{\tan x} . \sec^2 x + \log \tan x\left(-\operatorname{cosec}^2 x\right)$$

$$= \left(\frac{\cos x}{\sin x} \cdot \frac{\cos x}{\sin x} \cdot \frac{1}{\cos^2 x} - \operatorname{cosec}^2 x \log \tan x\right)$$

$$\therefore \frac{dy}{dx} = y\left[\operatorname{cosec}^2 x - \operatorname{cosec}^2 x \cdot \log \tan x\right]$$

$$= (\tan x)^{\cot x} \cdot \operatorname{cosec}^2 x[1 - \log \tan x] \qquad 1$$

7. $x = e^{2t}\cos t,\ y = e^{2t}\ .\sin t$

$$\frac{dx}{dt} = e^{2t}\ .\ (-\sin t) + \cos t\ .\ e^{2t}\ .\ 2$$

$$= e^{2t}[2\cos t - \sin t]$$ $\frac{1}{2}$

$$\frac{dy}{dt} = e^{2t}\ .\ \cos t + \sin t\ .\ e^{2t}\ .\ 2$$

$$= e^{2t}[2\sin t + \cos t]$$ $\frac{1}{2}$

$$\therefore \frac{dy}{dx} = \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{2\sin t + \cos t}{2\cos t - \sin t}$$ 1

8. Let $I = \int \tan^{-1} x\ .\ dx$

$$= \int \tan^{-1} x\ .\ 1dx$$

$$= \tan^{-1} x\ .\int 1\ dx - \int\left[\frac{d}{dx}\tan^{-1} x \int (1dx)\right]dx + c$$ 1

$$= \tan^{-1} x\ .x - \int \frac{1}{1+x^2}\ .\ x\ dx + c$$

$$= x\tan^{-1} x - \frac{1}{2}\log\left(1 + x^2\right) + c$$ 1

9. $\int \frac{dx}{9x^2 - 1} = \int \frac{dx}{(3x^2) - 1}$ 1

$$= \frac{1}{2} \cdot \log \frac{3x-1}{3x+1} \times \frac{1}{3} + c$$

$$= \frac{1}{6} \log \frac{3x-1}{3x+1} + c$$ 1

10. $(x^2 + xy)\, dy = -(3xy + y^2)\, dx$

$\Rightarrow \frac{dy}{dx} = -\frac{3xy + y^2}{x^2 + xy}$ .... (i)

Put $y = vx$

$\frac{dy}{dx} = v + x\frac{dv}{dx}$

$\therefore v + x\frac{dv}{dx} = -\frac{3x \cdot vx + v^2x^2}{x^2 + x \cdot vx}$

$\Rightarrow x\frac{dv}{dx} = -\frac{3v + v^2}{1+v} - v$

$$\Rightarrow x\frac{dv}{dx} = -\frac{-3v - v^2 - v - v^2}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{-4v - 2v^2}{1+v}$$ 1

$$\Rightarrow \int \frac{1+v}{2v+v^2}\,dv = -2\int \frac{dx}{x} + \log c$$

$$\Rightarrow \frac{1}{2}\log\left(2v+v^2\right) = -2\log x + \log c$$

$$\Rightarrow \log\sqrt{2v+v^2} = \log\frac{c}{x^2}$$

$$\Rightarrow \sqrt{\frac{2y}{x} + \frac{y^2}{x^2}} = \frac{c}{x^2}$$

$$\Rightarrow \sqrt{\frac{2xy+y^2}{x^2}} = \frac{c}{x^2}$$

$$\Rightarrow \frac{2xy+y^2}{x^2} = \frac{c^2}{x^4} \Rightarrow x^2 y(2x+y) = c$$ 1

11. $P(A) = \frac{3}{8}$, $P(B) = \frac{6}{10}$ 1

$P(AB) = P(A)\ .\ P(B)$

$$= \frac{3}{8} \times \frac{6}{10} = \frac{9}{40}$$ 1

SECTION – C

12. L. H. S. $= \tan^{-1}\dfrac{\sqrt{1+x^2}-1}{x}$

$= \tan^{-1}\dfrac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}$ put $x = \tan\theta$ 1

$= \tan^{-1}\dfrac{\sec\theta-1}{\tan\theta}$ 1

$= \tan^{-1}\left[\dfrac{1+\cos\theta}{\sin\theta}\right] = \tan^{-1}\left[\dfrac{2\sin^2\frac{\theta}{2}}{2\sin\frac{\theta}{2}\cos\frac{\theta}{2}}\right]$ 1

$= \tan^{-1}\left(\tan\dfrac{\theta}{2}\right) = \dfrac{\theta}{2} = \dfrac{1}{2}\tan^{-1}x =$ R. H. S. 1

13. $f(x) = |x-2| = \begin{cases} x-2 & , \ x \geq 2 \\ -(x-2) & , \ x < 2 \end{cases}$

$f(2) = 0$, R.H.L. $= \lim\limits_{x\to 2^+} f(x) = \lim\limits_{h\to 0}(2+h-2) = 0$

L. H. L. $= \lim\limits_{x\to 2^-} f(x) = \lim\limits_{h\to 0^-} -[2-h-2] = 0$

$\therefore f(x)$ is continuous at $x = 2$ 2

R. H. D. $= \lim_{x \to 2^+} \frac{(x-2)-0}{x-2} = 1$

L. H. D. $= \lim_{x \to 2^-} \frac{-(x-2)-0}{x-2} = -1$

$\therefore$ $f(x)$ is not differentiable at $x = 2$ 2

14. $x = a\sin^3 t,\ y = b\cos^3 t$

$$\frac{dx}{dt} = 3a\sin^2 t\cos t$$

$$\frac{dy}{dt} = -3b\cos^2 t\sin t$$

$$\therefore \frac{dy}{dx} = \frac{-3b\cos^2 t\sin t}{3a\sin^2 . t\cos t} = -\frac{b}{a}\frac{\cos t}{\sin t}$$ 1

$\therefore$ equation of tangent at $\left(a\sin^3 t, b\cos^3 t\right)$ is :

$$\left(y - b\cos^3 t\right) = -\frac{b}{a}\frac{\cos t}{\sin t}\left(x - a\sin^3 t\right)$$ 1

$$\Rightarrow \frac{y}{b\cos t} - \frac{b\cos^3 t}{b\cos t} = -\frac{x}{a\sin t} + \frac{a\sin^3 t}{a\sin t}$$

$$\Rightarrow \frac{x}{a\sin t} - \frac{y}{b\cos t} = \sin^2 t + \cos^2 t$$ 1

$$\Rightarrow \frac{x}{a\sin t} + \frac{y}{b\cos t} = 1$$ 1

15. $X = 0, 1, 2, 3, 4$

$p = \frac{1}{2}, q = \frac{1}{2}$ $\frac{1}{2}$

$P(X = 0) = {}^4C_0\left(\frac{1}{2}\right)^0 . \left(\frac{1}{2}\right)^4 = \frac{1}{16}$ $\frac{1}{2}$

$P(X = 1) = {}^4C_1\left(\frac{1}{2}\right)^1 . \left(\frac{1}{2}\right)^3 = \frac{4}{16} = \frac{1}{4}$ $\frac{1}{2}$

$P(X = 2) = {}^4C_2\left(\frac{1}{2}\right)^2 . \left(\frac{1}{2}\right)^2 = \frac{6}{16} = \frac{3}{8}$ $\frac{1}{2}$

$P(X = 3) = {}^4C_3\left(\frac{1}{2}\right)^3 . \left(\frac{1}{2}\right)^1 = \frac{4}{16} = \frac{1}{4}$ $\frac{1}{2}$

$P(X = 4) = {}^4C_4\left(\frac{1}{2}\right)^4 . \left(\frac{1}{2}\right)^0 = \frac{1}{16}$ $\frac{1}{2}$

X	0	1	2	3	4
$P(X)$	$\frac{1}{16}$	$\frac{1}{4}$	$\frac{3}{8}$	$\frac{1}{4}$	$\frac{1}{16}$

1

16. Let $A \equiv (1, 2, 3)$, $B \equiv (2, +5, -1)$, $C \equiv (-1, 1, 2)$

$\vec{a} = \hat{i} + 2\hat{j} + 3\hat{k}$, $\vec{b} = 2\hat{i} + 5\hat{j} - \hat{k}$, $\vec{c} = -\hat{i} + \hat{j} + 2\hat{k}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = -\hat{i} + \hat{j} + 2\hat{k} - 2\hat{i} - 5\hat{j} + \hat{k}$

$= -3\hat{i} - 4\hat{j} + 3\hat{k}$ 1

$\overrightarrow{BA} = \vec{a} - \vec{b} = -\hat{i} - 3\hat{j} + 4\hat{k}$ 1

Area of $\Delta ABC = \frac{1}{2} | \overrightarrow{BC} \times \overrightarrow{BA} |$

$$= \overrightarrow{BC} \times \overrightarrow{BA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ -3 & -4 & 3 \\ -1 & -3 & 4 \end{vmatrix}$$

$= \hat{i}(-16+9) - \hat{j}(-12+3) + \hat{k}(9-4)$

$= -7\hat{i} + 9\hat{j} + 5\hat{k}$ 1

$\therefore$ Area of $\Delta ABC = \frac{1}{2}\sqrt{49+81+25}$

$= \frac{1}{2}\sqrt{155}$ sq. units 1

SECTION – D

17. $x + 2y - 3z = -4$

$2x + 3y + 2z = 2$

$3x - 3y - 4z = 11$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$ 1

$AX = B$

$\Rightarrow X = A^{-1}B$

$$|A| = \begin{vmatrix} 1 & 2 & -3 \\ 2 & 3 & 2 \\ 3 & -3 & -4 \end{vmatrix} = 1(-12+6) - 2(-8-6) - 3(-6-9)$$

$$= -6 + 28 + 45 = 67 \neq 0$$ 1

$A_{11} = -6,\ A_{12} = 14,\ A_{13} = -15$

$A_{21} = 17,\ A_{22} = 5,\ A_{23} = 9$

$A_{31} = 13,\ A_{32} = -8,\ A_{33} = -1$

$$Adj\ A = \begin{bmatrix} -6 & 14 & -15 \\ 17 & 5 & 9 \\ 13 & -8 & -1 \end{bmatrix}' = \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix}$$ 2

$$\therefore X = A^{-1}B = \frac{1}{67} \begin{bmatrix} -6 & 17 & 13 \\ 14 & 5 & -8 \\ -15 & 9 & -1 \end{bmatrix} \begin{bmatrix} -4 \\ 2 \\ 11 \end{bmatrix}$$ 1

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 24+34+143 \\ -56+10-88 \\ 60+18-11 \end{bmatrix} = \frac{1}{67}\begin{bmatrix} 201 \\ -134 \\ 67 \end{bmatrix} = \begin{bmatrix} 3 \\ -2 \\ 1 \end{bmatrix}$$

$x = 3,\ y = -2,\ z = 1$ 1

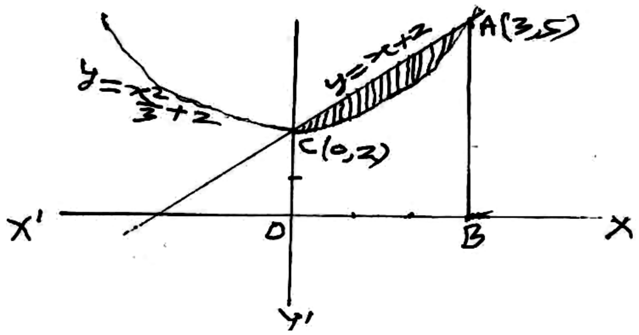
18. $y = x + 2$ ..... (i)

$y = \frac{x^2}{3} + 2$ ..... (ii)

From (i) and (ii), we get

$x = 0, 3$

when $x = 0,\ y = 2$ and when $x = 3,\ y = 5$ 1



2

Required Area $= \int_0^3 [(y \text{ of line}) - (y \text{ of parabola})]\, dx$

1

$$= \int_0^3 \left[(x+2) - \left(\frac{x^2}{3} + 2\right)\right] dx$$

$$= \int_0^3 \left[x - \frac{x^2}{3} \right] dx$$ 1

$$= \left[\frac{x^2}{2} - \frac{x^3}{9} \right]_0^3 = \frac{9}{2} - 3 = \frac{3}{2} \text{ sq. units}$$

1

OR

Let $I = \int_0^\pi \frac{x}{1+\sin^2 x}\, dx$ .... (i)

$\therefore\ I = \int_0^\pi \frac{\pi - x}{1+\sin^2(\pi - x)}\, dx$

$\therefore\ I = \int_0^\pi \frac{\pi - x}{1+\sin^2 x}\, dx$ .... (ii) 1

Adding (i) and (ii), we get

$2I = \int_0^\pi \frac{\pi}{1+\sin^2 x}\, dx$ 1

$\therefore\ I = \frac{\pi}{2}\int_0^\pi \frac{dx}{1+\sin^2 x} = \frac{\pi}{2} . 2 \int_0^{\pi/2} \frac{\sec^2 x\, dx}{\sec^2 x + \tan^2 x}$ 1

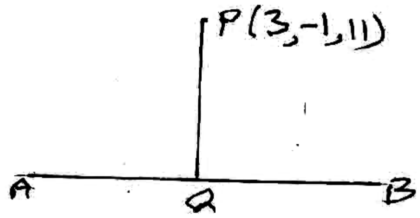
$\Rightarrow I = \pi \int_0^{\pi/2} \frac{\sec^2 x\, dx}{1 + 2\tan^2 x}$ 1

put $\tan x = t \Rightarrow \sec^2 x\, dx = dt$

$\therefore\ I = \pi\int_0^\infty \frac{dt}{1+\left(\sqrt{2}t\right)^2} = \frac{\pi}{\sqrt{2}}\left[\tan^{-1}\left(\sqrt{2}t\right)\right]_0^\infty$ 1

$= \frac{\pi}{\sqrt{2}}\left[\frac{\pi}{2}-0\right] = \frac{\pi^2}{2\sqrt{2}}$ 1

19. Let $P(3, -1, 11)$ be the given point and Q be the foot of perpendicular to the given line AB,



$\frac{x}{2} = \frac{y-2}{3} = \frac{z-3}{4} = \lambda$

$x = 2\lambda$, $y = 3\lambda + 2$, $z = 4\lambda + 3$ be the 1

General point of line AB say Q

$\therefore\ Q\,(2\lambda, 3\lambda + 2, 4\lambda + 3)$

Direction ratios of line PQ are

$2\lambda - 3, 3\lambda + 3, 4\lambda - 8$ 1

and D. R's of line AB are 2, 3, 4 1

Now $AB \perp PQ$

$\therefore 2(2\lambda - 3) + 3\,(3\lambda + 3) + 4(4\lambda - 8) = 0$ 1

$\Rightarrow 29\lambda = 29 \Rightarrow \lambda = 1$

$\therefore\ Q \equiv (2, 5, 7)$ 1

equation of PQ is

$$\frac{x-3}{2-3}=\frac{y+1}{5+1}=\frac{z-11}{7-11}$$

$$\Rightarrow \frac{x-3}{-1}=\frac{y+1}{6}=\frac{z-11}{-4}$$ 1

OR

Equation of plane passing through (2, 1, 0) is :

$a(x-2)+b(y-1)+c(z-0)=0$ .... (i) 1

plane also passing through (3, –2, –2) and (3, 1, 7)

$a-3b-2c=0$ .... (ii) 1

$a+0b+7c=0$ .... (iii) 1

$$\frac{a}{-21-0}=\frac{b}{-2-7}=\frac{c}{0+3}$$

$$\Rightarrow \frac{a}{7}=\frac{b}{3}=\frac{c}{-1}=\lambda \text{ (say)}$$

$a=7\lambda,\ b=3\lambda,\ c=-\lambda$ 1

$\therefore$ Required plane is

$7\lambda(x-2)+3\lambda(y-1)-\lambda(z-0)=0$ 1

$\Rightarrow 7x+3y-z-17=0$ 1

20. For tabulation 2

For graphical representation 2

Minimum $z = 134$ 1

at $x = 3,\ y = 8$ 1

SET – B

SECTION – A

1. (i) $(fog)\ (x) = fog\ (x)$

$$= f\left(e^{x}\right) = \log\left(1 + e^{x}\right)$$ **Ans. (B)** 1

(ii) Let $\sin^{-1}\dfrac{8}{17} = \theta \Rightarrow \sin\theta = \dfrac{8}{17}$

$$\therefore \cos\left(\sin^{-1}\frac{8}{17}\right) = \cos\theta = \sqrt{1 - \sin^{2}\theta}$$

$$= \sqrt{1 - \left(\frac{8}{17}\right)^{2}} = \sqrt{\frac{289 - 64}{289}}$$

$$= \sqrt{\frac{225}{289}} = \frac{15}{17}$$

Ans. (C) 1

(iii) $X = A + B = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -1 & 2 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 0 & 6 \\ 3 & 7 \end{bmatrix}$

Ans. (A) 1

(iv) $x^2 = 36 = 36 - 36 \Rightarrow x^2 = 36$

$\therefore\ x = \pm 6$ **Ans. (B)** 1

(v) $\frac{d}{dx}\left(\tan^3 x\right) = 3\tan^2 x \,.\, \sec^2 x$ **Ans. (C)** 1

(vi) $-1 \leq \sin 2x \leq 1$

Maximum $\sin 2x = 1$

$\Rightarrow 2x = \frac{\pi}{2} \Rightarrow x = \frac{\pi}{4}$ **Ans. (B)** 1

(vii) $x = a\cos^3 \theta,\ \ y = a\sin^3 \theta$

$\frac{dx}{d\theta} = -3a\cos^2 \theta \sin\theta$

$\frac{dy}{d\theta} = 3a\sin^2 \theta \cos\theta$

$\therefore \frac{dy}{dx} = \frac{3a\sin^2 \theta \,.\cos\theta}{-3a\cos^2 \theta \,.\, \sin\theta} = -\tan\theta$

at $\theta = \frac{\pi}{4}, \frac{dy}{dx} = -\tan\frac{\pi}{4} = -1$

$\therefore$ slope of normal = 1 **Ans. (A)** 1

(viii) $\int \cos^2 x \, dx = \int \frac{1+\cos 2x}{2} dx$

$= \frac{1}{2}\left[x + \frac{\sin 2x}{2}\right] + c$ **Ans. (A)** 1

(ix) $I = \int \frac{3x^5}{1+x^{12}} dx$

Put $x^6 = t, \quad 6x^5 \, dx = dt$

$= \frac{1}{2}\int \frac{dt}{1+t^2}$

$= \frac{1}{2}\tan^{-1} t + c = \frac{1}{2}\tan^{-1} x^6 + c$ **Ans. (B)** 1

(x) Degree = 1 **Ans. (A)** 1

(xi) $\frac{dy}{dx} = e^{-x}$

Integrate, y = $-e^{-x} + c$ **Ans. (C)** 1

(xii) $P(B/A) = \frac{P(A \cap B)}{P(A)} = \frac{\frac{4}{13}}{\frac{7}{13}} = \frac{4}{7}$ **Ans. (A)** 1

(xiii) $P(A)\ P(B/A) = \frac{10}{25} \times \frac{15}{24} = \frac{1}{4}$ **Ans. (B)** 1

(xiv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A) \,.\, P(B)$

$\Rightarrow 0.7 = 0.6 + P(B)[1 - P(A)]$

$\Rightarrow 0.7 = 0.6 + P(B)[1 - 0.6]$

$\Rightarrow 0.1 = 0.4P(B)$

$\Rightarrow P(B) = \frac{1}{4}$. **Ans. (C)** 1

(xv) $\vec{a} \,.\, \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$

$= ab \cos\theta$

$\Rightarrow \vec{a} \,.\, \vec{b} = ab$ **Ans. (C)** 1

(xvi) $l = \cos\theta,\ m = \cos\theta,\ n = \cos\theta,$

$$l^2 + m^2 + n^2 = 1$$

$$\Rightarrow 3\cos^2\theta = 1 \Rightarrow \cos^2\theta = \frac{1}{3}$$

$$\Rightarrow \cos\theta = \pm\frac{1}{\sqrt{3}}$$ **Ans. (D)** 1

SECTION – B

2. $f(1) = \frac{1+1}{2} = 1$

$f(2) = \frac{2}{2} = 1$

$f(3) = \frac{3+1}{2} = 2$

$f(4) = \frac{4}{2} = 2$ 1

different elements in domain have same images in co-domain

$\therefore f$ is not one-one 1

3. Let $\tan^{-1} x = \theta$ .... (i)

$$\Rightarrow x = \tan\theta = \cot\left(\frac{\pi}{2} - \theta\right)$$

$\Rightarrow \cot^{-1} x = \frac{\pi}{2} - \theta$ .... (ii) 1

adding (i) and (ii), we get

$\tan^{-1} x + \cot^{-1} x = \theta + \frac{\pi}{2} - \theta$

$\Rightarrow \tan^{-1} x + \cot^{-1} x = \frac{\pi}{2}$ 1

4. $A^2 = A.A = \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix}$

$= \begin{bmatrix} 4-3 & 6+6 \\ -2-2 & -3+4 \end{bmatrix} = \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix}$ 1

$f(A) = A^2 - 4A + 7I$

$= \begin{bmatrix} 1 & 12 \\ -4 & 1 \end{bmatrix} - 4 \begin{bmatrix} 2 & 3 \\ -1 & 2 \end{bmatrix} + 7 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$= \begin{bmatrix} 1-8+7 & 12-12 \\ -4+4 & 1-8+7 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$ 1

5. Area of triangle $= \frac{1}{2} \begin{vmatrix} 4 & 2 & 1 \\ 4 & 5 & 1 \\ -2 & 2 & 1 \end{vmatrix}$ 1

$= \frac{1}{2}$ [4(5 – 2) – 2(4 + 2) + 1(8 + 10)]

$= \frac{1}{2}$ [12 – 12 + 18]

= 9 sq. units 1

6. Let $y = (\sin x)^{\log x}$

$\Rightarrow \log y = \log x \,.\, \log(\sin x)$ 1

diff. w. r. t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \log x \,.\, \frac{1}{\sin x}\,.\cos x + \log(\sin x)\,.\,\frac{1}{x}$$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^{\log x}\left[\log x \,.\, \cot x + \frac{\log(\sin x)}{x}\right]$$ 1

7. $x = \cos 2\theta + 2\cos\theta,\ y = \sin 2\theta - 2\sin\theta$

$$\frac{dx}{d\theta} = -\sin 2\theta - 2\sin\theta,\ \frac{dy}{d\theta} = 2\cos 2\theta - 2\cos\theta$$ 1

$$\therefore \frac{dy}{dx} = \frac{2\cos 2\theta - 2\cos\theta}{-2\sin 2\theta - 2\sin\theta}$$

$$= \frac{\cos 2\theta - \cos\theta}{-\sin 2\theta - \sin\theta}$$

$$= \frac{2\sin\frac{3\theta}{2}\sin\left(\frac{-\theta}{2}\right)}{-2\sin\frac{3\theta}{2}\cos\frac{\theta}{2}} = \frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}} = \tan\frac{\theta}{2}$$ 1

8. Let $I = \int \sin^{-1} x \,.\, 1dx = \sin^{-1} x \int 1dx -$

$$\int\left[\frac{d}{dx}\sin^{-1} x \int 1dx\right] dx$$

$$= \sin^{-1} x \,.\, x - \int \frac{1}{\sqrt{1-x^2}} \,.\, x\, dx + c$$

put $1 - x^2 = t^2 \Rightarrow -2x\, dx = 2t\, dt$ 1

$$I = x \sin^{-1} x + \int \frac{t\,.\,dt}{t} + c$$

$$= x \sin^{-1} x + t + c$$

$$= x \sin^{-1} x + \sqrt{1-x^2} + c$$ 1

9. $\int \frac{dx}{1-4x^2} = \int \frac{dx}{1-(2x)^2}$ 1

$$= \frac{1}{2x^2} \,.\, \log \frac{1+2x}{1-2x} + c$$

$$= \frac{1}{4} \log \frac{1+2x}{1-2x} + c$$ 1

10. $2xy\, dy = -\left(x^2 + y^2\right) dx$

$$\Rightarrow \frac{dy}{dx} = -\frac{x^2 + y^2}{2xy}$$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\therefore v + x\frac{dv}{dx} = -\frac{x^2 + v^2x^2}{2x \,.\, vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = -\left[\frac{1+v^2}{2v}\right]$$

$$\Rightarrow x\frac{dv}{dx} = -v - \frac{1+v^2}{2v}$$

$$= \frac{-2v^2 - 1 - v^2}{2v}$$

$$\Rightarrow \int \frac{2v}{1+3v^2}\,dv = -\int \frac{dx}{x} + c$$ 1

put $1 + 3v^2 = t \Rightarrow 6v\,dv = dt$

$$\Rightarrow \frac{1}{3}\int \frac{dt}{t} = -\log x + \log c$$

$$\Rightarrow \frac{1}{3}\log t = \log\frac{c}{x}$$

$$\Rightarrow t^{1/3} = \frac{c}{x}$$

$$\Rightarrow \left(1 + 3v^2\right)^{1/3} = \frac{c}{x}$$

$$\Rightarrow \left(1+\frac{3y^2}{x^2}\right)^{1/3} = \frac{c}{x}$$

$$\Rightarrow \left(x^2+3y^2\right)^{1/3} . x^{1/3} = c$$

$$\Rightarrow x\left(x^2+3y^2\right) = k$$ 1

11. $P(AB) = P(A)P(B)$

$$= \frac{5}{8}\times\frac{4}{10}$$ 1

$$= \frac{1}{4}$$ 1

SECTION – C

12. L. H. S. $= \tan^{-1} \dfrac{\sqrt{1+x^2}+\sqrt{1-x^2}}{\sqrt{1+x^2}-\sqrt{1+x^2}}$

Put $x^2 = \cos 2\theta$

$$= \tan^{-1} \frac{\sqrt{1+\cos 2\theta}+\sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta}-\sqrt{1-\cos 2\theta}}$$ 1

$$= \tan^{-1}\left[\frac{\sqrt{2\cos^2\theta}+\sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta}-\sqrt{2\sin^2\theta}}\right]$$ 1

$$= \tan^{-1} \frac{\cos\theta + \sin\theta}{\cos\theta - \sin\theta}$$ 1

$$= \tan^{-1} \frac{1+\tan\theta}{1-\tan\theta} = \tan^{-1} \tan\left(\frac{\pi}{4} + \theta\right)$$

$$= \frac{\pi}{4} + \theta = \frac{\pi}{4} + \frac{1}{2}\cos^{-1} x^2 = \text{R. H. S.}$$ 1

13. $f(x) = | x - 1 | \; | x + 1 |$

$$= \begin{cases} -(x-1)-(x+1) = -2x & , \quad x < -1 \\ -(x-1)+(x+1) = 2 & , \quad -1 \le x < 1 \\ x-1+x+1 = 2x & , \quad x \ge 1 \end{cases}$$

1

at $x = -1$

$$\text{R. H. D.} = \lim_{x \to -1^+} \frac{f(x) - f(-1)}{x - (-1)}$$

$$= \lim_{x \to -1^+} \frac{2-2}{x+1} = 0$$ 1

$$\text{L. H. D.} = \lim_{x \to -1^-} \frac{f(x) - f(-1)}{x - (-1)}$$

$$= \lim_{x \to -1^-} \frac{-2x-2}{x+1} = \lim_{x \to -1^-} \frac{-2(x+1)}{x+1}$$

$$= -2$$ 1

L. H. D. $\neq$ R. H. D.

$\therefore f(x)$ is not differentiable at $x = -1$ 1

14. $x = 1 - \cos\theta,\ \ y = \theta - \sin\theta$

at $\theta = \dfrac{\pi}{4},\ x = 1 - \cos\dfrac{\pi}{4} = 1 - \dfrac{1}{\sqrt{2}} = \dfrac{\sqrt{2}-1}{\sqrt{2}}$

$\therefore\ x = \dfrac{2-\sqrt{2}}{2}$

$y = \dfrac{\pi}{4} - \sin\dfrac{\pi}{4} = \dfrac{\pi}{4} - \dfrac{1}{\sqrt{2}} = \dfrac{\pi}{4} - \dfrac{\sqrt{2}}{2} = \dfrac{\pi - 2\sqrt{2}}{4}$ 1

$\dfrac{dx}{d\theta} = \sin\theta, \dfrac{dy}{d\theta} = 1 - \cos\theta$

$\therefore\ \dfrac{dy}{dx} = \dfrac{1-\cos\theta}{\sin\theta}$

at $\theta = \dfrac{\pi}{4}, \dfrac{dy}{dx} = \dfrac{1-\cos\dfrac{\pi}{4}}{\sin\dfrac{\pi}{4}} = \dfrac{1-\dfrac{1}{\sqrt{2}}}{\dfrac{1}{\sqrt{2}}}$ 1

$= \sqrt{2} - 1$ 1

$\therefore$ Equation of tangent at $\theta = \dfrac{\pi}{4}$

$y = \dfrac{\pi - 2\sqrt{2}}{4} = \left(\sqrt{2}-1\right)\left(x - \dfrac{2-\sqrt{2}}{2}\right)$

$y = \left(\sqrt{2}-1\right)x + 2 - 2\sqrt{2} + \dfrac{\pi}{4}$ 1

15. $S = \{(3, 6), (4, 5), (5, 4), (6, 3)\}$ 1

$$p = \frac{4}{36} = \frac{1}{9}, \; q = \frac{8}{9}$$ 1

$X = 0, 1, 2$

$$P(X = 0) = {}^2C_0\left(\frac{1}{9}\right)^0 \cdot \left(\frac{8}{9}\right)^2 = \frac{64}{81}$$ $\frac{1}{2}$

$$P(X = 1) = {}^2C_1\left(\frac{1}{9}\right)^1 \cdot \left(\frac{8}{9}\right)^1 = \frac{16}{81}$$ $\frac{1}{2}$

$$P(X = 2) = {}^2C_2\left(\frac{1}{9}\right)^2 \cdot \left(\frac{8}{9}\right)^0 = \frac{1}{81}$$ $\frac{1}{2}$

X	0	1	2
$P(X)$	$\frac{64}{81}$	$\frac{16}{81}$	$\frac{1}{81}$

$\frac{1}{2}$

16. $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \; \vec{b} = \hat{j} - \hat{k}$

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$

$$\vec{a} \times \vec{c} = \vec{b} \Rightarrow \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 1 \\ c_1 & c_2 & c_3 \end{vmatrix} = \hat{j} - \hat{k}$$ 1

$\hat{i}(c_3 - c_2) - \hat{j}(c_3 - c_1) + \hat{k}(c_2 - c_1) = \hat{j} - \hat{k}$

$c_3 - c_2 = 0$ .... (i)

$c_1 - c_3 = 1$ .... (ii)

$c_1 - c_2 = 1$ .... (iii)

$\vec{a} \,.\, \vec{c} = 3$

$c_1 + c_2 + c_3 = 3$ .... (iv) 1

From (i) and (iv), we get

$c_1 + 2c_3 = 3$ .... (v)

(5) – (2) $3c_3 = 2 \Rightarrow c_3 = \dfrac{2}{3}$

$\therefore\; c_2 = \dfrac{2}{3},\;\; c_1 = \dfrac{5}{3}$ 1

$\therefore\; \vec{c} = \dfrac{5}{3}\hat{i} + \dfrac{2}{3}\hat{j} + \dfrac{2}{3}\hat{k}$ 1

SECTION – D

17. $8x + 4y + 3z = 19$

$2x + y + z = 5$

$x + 2y + 2z = 7$

$$\begin{bmatrix} 8 & 4 & 3 \\ 2 & 1 & 1 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 19 \\ 5 \\ 7 \end{bmatrix}$$

$AX = B$

$\Rightarrow X = A^{-1}B$

$|A| = 8(2-2) - 4(4-1) + 3\ (4-1)$

$= 0 - 12 + 9 = -3 \neq 0$ 1

$A_{11} = 0,\ A_{12} = -3,\ A_{13} = 3$

$A_{21} = -2,\ A_{22} = 13,\ A_{23} = -12$

$A_{31} = 1,\ A_{32} = -2,\ A_{33} = 0$ 1

$$Adj\ A = \begin{bmatrix} 0 & -3 & 3 \\ -2 & 13 & -12 \\ 1 & -2 & 0 \end{bmatrix}' = \begin{bmatrix} 0 & -2 & 1 \\ -3 & 13 & -2 \\ 3 & -12 & 0 \end{bmatrix}$$ 1

$\therefore X = A^{-1}B$

$$= -\frac{1}{3}\begin{bmatrix} 0 & -2 & 1 \\ -3 & 13 & -2 \\ 3 & -12 & 0 \end{bmatrix}\begin{bmatrix} 19 \\ 5 \\ 7 \end{bmatrix}$$ 1

$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} 0-10+7 \\ -57+65-14 \\ 57-60+0 \end{bmatrix} = -\frac{1}{3}\begin{bmatrix} -3 \\ -6 \\ -3 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$ 1

$x = 1,\ y = 2,\ z = 1$ 1

18. $x^2 = 4y$ ..... (i)

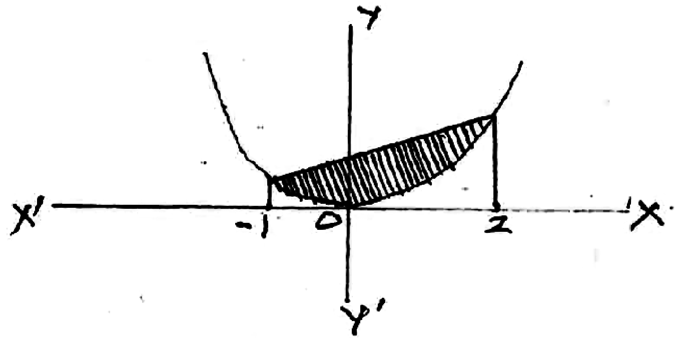
$x = 4y - 2$ ..... (ii)

From (i) and (ii),

$x = x^2 - 2$

$\Rightarrow x^2 - x - 2 = 0$

$\Rightarrow (x-2)\ (x+1) = 0 \Rightarrow x = -1,\ 2$ 1



2

Required Area $= \int_{-1}^{2} [(y \text{ of line})\ dx - \int_{-1}^{2} [(y \text{ of parabola})\ dx$

$= \frac{1}{4}\int_{-1}^{2} (x+2)\ dx - \frac{1}{4}\int_{-1}^{2} x^2\ dx$ 1

$= \frac{1}{4}\left[\frac{x^2}{2} + 2x\right]_{-1}^{2} - \frac{1}{4}\left[\frac{x^3}{3}\right]_{-1}^{2}$

$= \frac{1}{4}\left[(2+4) - \left(\frac{1}{2} - 2\right)\right] - \frac{1}{4}\left[\frac{8}{3} + \frac{1}{3}\right]$ 1

$= \frac{15}{8} - \frac{3}{4} = \frac{9}{8}$ sq. units 1

OR

Let $I = \int_0^{\pi} \frac{x}{4 - \cos^2 x} dx$ .... (i)

$\therefore \; I = \int_0^{\pi} \frac{\pi - x}{4 - \cos^2(\pi - x)} dx$ 1

$\therefore \; I = \int_0^{\pi} \frac{\pi - x}{4 - \cos^2 x} dx$ .... (ii) 1

Adding (i) and (ii), we get

$2I = \pi \int_0^{\pi} \frac{dx}{4 - \cos^2 x}$

$= 2\pi \int_0^{\pi/2} \frac{\sec^2 x dx}{4\sec^2 x - 1}$ 1

$= 2\pi \int_0^{\pi/2} \frac{\sec^2 x dx}{3 + 4\tan^2 x}$

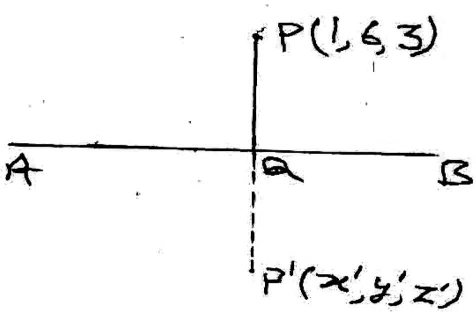
Put $\tan x = t, \; \sec^2 x \, dx = dt$

$\therefore \; I = \frac{\pi}{4} \int_0^{\infty} \frac{dt}{\left(\frac{\sqrt{3}}{2}\right)^2 + t^2}$ 1

$$= \frac{\pi}{4} \cdot \frac{2}{\sqrt{3}} \left[\tan^{-1} \frac{2t}{\sqrt{3}} \right]_0^{\infty} = \frac{\pi}{2\sqrt{3}} \left[\frac{\pi}{2} - 0 \right]$$ 1

$$\therefore I = \frac{\pi^2}{4\sqrt{3}}$$ 1

19. Let $P(1, 6, 3)$ and line AB is $\frac{x}{1} = \frac{y-1}{2} = \frac{z-2}{3} = k$



1

$x = k$, $y = 2k + 1$, $z = 3k + 2$ be general pt. on AB

Let Q $(k, 2k + 1, 3k + 2)$

$\therefore$D. R's of line PQ are

$k - 1, 2k - 5, 3k - 1$ 1

and DR's of line AB are 1, 2, 3

$AB \perp PQ$

$1 - (k - 1) + 2(2k - 5) + 3(3k - 1) = 0$ 1

$\Rightarrow 14k = 14 \Rightarrow k = 1$

$\therefore\ Q \equiv (1, 3, 5)$ 1

$P'(x', y', z')$ be the image of $P(1, 6, 3)$ and $Q(1, 3, 5)$ is Mid pt. of PP'

$\therefore 1 = \frac{x'+1}{2}, 3 = \frac{6+y'}{2}, 5 = \frac{3+z'}{2}$ 1

$\Rightarrow x' = 1, y' = 0, z' = 7$

$\therefore$ Image of $P \equiv (1, 0, 7)$ 1

OR

Equation of plane passing through (0, 1, 1) is :

$a(x - 0) + b(y - 1) + c(z - 1) = 0$ .... (i) 1

plane passing (1, 1, 2) and (–1, 2, –2)

$\therefore a + 0b + c = 0$ .... (ii) 1

$-a + b - 3c = 0$ .... (iii) 1

$$\frac{a}{0-1} = \frac{b}{-1+3} = \frac{c}{1-0}$$

$\Rightarrow \frac{a}{1} = \frac{b}{-2} = \frac{c}{-1} = \lambda$ 1

$\Rightarrow \lambda(x - 0) - 2\lambda(y - 1) - \lambda(z - 1) = 0$ 1

$\Rightarrow x - 2y - z + 3 = 0$ 1

20. For tabulation 2

For graphical representation 2

Minimum $z = -12$ 1

at $x = 4,\ y = 0$ 1

SET – C

SECTION – A

1. (i) $fog\,(x) = f\left(x^2\right)$

$$= \frac{x^2+1}{x^2+2}$$ **Ans. (B)** 1

(ii) Let $\tan^{-1}\frac{3}{4} = \theta \Rightarrow \tan\theta = \frac{3}{4}$

$$\therefore \cos\left(\tan^{-1}\frac{3}{4}\right) = \cos\theta = \frac{1}{\sec\theta}$$

$$= \frac{1}{\sqrt{1+\tan^2\theta}} = \frac{1}{\sqrt{1+\frac{9}{16}}} = \frac{4}{5}$$

Ans. (A) 1

(iii) $X = \begin{bmatrix} 2 & 3 \\ 3 & 1 \end{bmatrix} - \begin{bmatrix} 1 & 5 \\ 0 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -2 \\ 3 & -3 \end{bmatrix}$ **Ans. (C)** 1

(iv) $4\,(5 - x) - 2(x + 1) = 0$

$\Rightarrow 20 - 4x - 2x - 2 = 0$

$\Rightarrow 6x = 18 \Rightarrow x = 3$ **Ans. (A)** 1

(v) $\frac{d}{dx}\left(\cos x^4\right) = -\sin x^4 \, . \, \frac{d}{dx}\left(x^4\right)$ 1

$= -4x^3 \sin x^4$ **Ans. (C)** 1

(vi) $f(x) = \cos x - \sin x$

$\Rightarrow f'(x) = -\sin x - \cos x$

For maximum or minimum $f'(0) = 0$

$-\sin x - \cos x = 0$

$-\sin x = \cos x$

$\Rightarrow \tan x = -1$

$\Rightarrow x = \frac{3\pi}{4}$ **Ans. (B)** 1

(vii) $x = a(\theta - \sin\theta)$, $y = a(1 - \cos\theta)$

$$\frac{dx}{d\theta} = a[1-\cos\theta], \frac{dy}{d\theta} = a\sin\theta$$

$$\therefore \frac{dy}{dx} = \frac{a\sin\theta}{a[1-\cos\theta]}$$

at $\theta = \frac{\pi}{2}, \frac{dy}{dx} = \frac{1}{1-0} = 1$ **Ans. (A)** 1

(viii) $\int \cos^2 \frac{x}{2}\, dx = \int \frac{1+\cos x}{2} dx$

$= \frac{1}{2}[x + \sin x] + c$ **Ans. (C)** 1

(ix) $\int \frac{x}{x^2+1}\, dx = \frac{1}{2}\int \frac{2x}{x^2+1}\, dx$

$= \frac{1}{2}\log(x^2+1) + c$ **Ans. (B)** 1

(x) Degree = 2

$\left(\frac{d^2y}{dx^2}\right)^2 = 1 + \frac{dy}{dx}$ **Ans. (C)** 1

(xi) $\frac{dy}{dx} = x^2 + \sin 3x$

Integrate w. r. t. x

$$y = \frac{x^3}{3} - \frac{1}{3}\cos 3x + c$$ **Ans. (C)** 1

(xii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore\ P(A \cap B) = P(A) + P(B) - P(A \cup B)$

$= 0.5 + 0.6 - 0.8 = 0.3$

$$\therefore P(A/B) = \frac{P(A \cap B)}{P(B)} = \frac{0.3}{0.6} = \frac{1}{2}$$ **Ans. (A)** 1

(xiii) $P(AB) = P(A)\ P(B/A)$

$$= \frac{13}{52} \times \frac{12}{51} = \frac{1}{4} \times \frac{4}{17} = \frac{1}{17}$$ **Ans. (C)** 1

(xiv) $P(A \cap B) = P(A) \cdot P(B)$

$$= \frac{3}{5} \times \frac{1}{5} = \frac{3}{25}$$ **Ans. (B)** 1

(xv) $\vec{a} \cdot \vec{b} = |\vec{a}| \cdot |\vec{b}| \cdot \cos\theta$

$$\Rightarrow \cos\theta = \frac{\vec{a} \cdot \vec{b}}{|\vec{a}| \cdot |\vec{b}|} = \frac{3}{\sqrt{3} \cdot 2} = \frac{\sqrt{3}}{2}$$

$\Rightarrow \theta = \frac{\pi}{6}$ **Ans. (D)** 1

(xvi) D. R's of line are 1 + 2, 2 – 4, 3 + 5

$\equiv 3, -2, 8$

$\therefore$ D. C's are $\frac{3}{\sqrt{77}}, \frac{-2}{\sqrt{77}}, \frac{8}{\sqrt{77}}$ **Ans. (B)** 1

SECTION – B

2. Let $x_1, x_2 \in N$

If x_1, x_2 are odd numbers, then

$f(x_1) = f(x_2) \Rightarrow x_1 + 1 = x_2 + 1$

$\Rightarrow x_1 = x_2$

If x_1, x_2 are even numbers, then

$f(x_1) = f(x_2) \Rightarrow x_1 - 1 = x_2 - 1$

$\Rightarrow x_1 = x_2$ 1

Also if x_1 is even, x_2 is odd

$\Rightarrow x_1 \neq x_2 \Rightarrow x_1 - 1 \neq x_2 + 1$

$\Rightarrow$ odd $\neq$ even $\Rightarrow f(x_1) \neq f(x_2)$

$\therefore f$ is one-one 1

3. $\sec^{-1} x + \text{cosec}^{-1} x = \frac{\pi}{2}$ To Prove

Let $\sec^{-1} x = \theta \Rightarrow \sec\theta = x$ ...... (i)

$\Rightarrow \text{cosec}\left(\frac{\pi}{2} - \theta\right) = x$

$\Rightarrow \frac{\pi}{2} - \theta = \text{cosec}^{-1} x$ ...... (ii) 1

From (i) and (ii), we get,

$\sec^{-1} x + \text{cosec}^{-1} x = \theta + \frac{\pi}{2} - \theta$

$= \frac{\pi}{2}$ 1

4. $A^2 = A.A$

$= \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1+4 & 2+2 \\ 2+2 & 4+1 \end{bmatrix} \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix}$ 1

$f(A) = A^2 - 2A - 3I$

$$= \begin{bmatrix} 5 & 4 \\ 4 & 5 \end{bmatrix} - 2\begin{bmatrix} 1 & 2 \\ 2 & 1 \end{bmatrix} - 3\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 5-2-3 & 4-4 \\ 4-4 & 5-2-3 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0 \qquad 1$$

5. Area of triangle $= \dfrac{1}{2}\begin{vmatrix} -3 & 1 & 1 \\ 2 & -4 & 1 \\ 5 & 1 & 1 \end{vmatrix}$ 1

$$= \frac{1}{2}[-3(-4-1) - 1(2-5) + 1(2+20)]$$

$$= \frac{1}{2}[15 + 3 + 22] = 20 \text{ sq. units} \qquad 1$$

6. Let $y = x^{\sin^{-1} x}$

taking log both side, we get

$\log y = \sin^{-1} x \,.\, \log x$ 1

diff. w. r. t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \sin^{-1} x \,.\, \frac{1}{x} + \log x.\, \frac{1}{\sqrt{1-x^2}}$$

$$\therefore \frac{dy}{dx} = x^{\sin^{-1} x}\left[\frac{\sin^{-1} x}{x} + \frac{\log}{\sqrt{1-x^2}}\right] \qquad 1$$

7. $x = a\,[\theta + \sin\theta]$, $y = a\,(1 - \cos\theta)$

$$\frac{dx}{d\theta} = a[1+\cos\theta],\ \frac{dy}{d\theta} = a[0+\sin\theta]$$

$$\frac{dy}{dx} = \frac{\frac{dy}{d\theta}}{\frac{dx}{d\theta}} = \frac{a\sin\theta}{a(1+\cos\theta)} = \frac{\sin\frac{\pi}{2}}{1+\cos\frac{\pi}{2}} = 1$$ 1

at $\theta = \frac{\pi}{2}, \frac{dy}{dx} = \frac{1+0}{1} = 1$ 1

8. Let $I = \int \cos^{-1} x \,.\, 1dx = \cos^{-1} x \int 1dx -$

$$\int \left[\frac{d}{dx}\cos^{-1} x \int 1dx \right] dx + c$$

$$= \cos^{-1} x \,.\, x + \int \frac{1}{\sqrt{1-x^2}} \,.\, x\,dx + c$$

put $1 - x^2 = t^2 \Rightarrow -2x\,dx = 2t\,dt$ 1

$$\therefore I = x\cos^{-1} x - \int \frac{t\,.\,dt}{t} + c$$

$$= x\cos^{-1} x - t + c$$

$$= x\cos^{-1} x - \sqrt{1-x^2} + c$$ 1

9. $\int \frac{dx}{9+25x^2} = \frac{1}{25}\int \frac{dx}{\left(\frac{3}{5}\right)^2 + x^2}$ 1

$= \frac{1}{25} \cdot \frac{5}{3} \tan^{-1} \frac{5x}{3} + c$

$= \frac{1}{15} \tan^{-1} \frac{5x}{3} + c$ 1

10. $\frac{dy}{dx} = \frac{x^2 - y^2}{xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2 - v^2 x^2}{x \,.\, vx}$

$\Rightarrow v + x\frac{dv}{dx} = \frac{1 - v^2}{v}$

$\Rightarrow x\frac{dv}{dx} = \frac{1 - v^2}{v} - v$

$x\frac{dv}{dx} = \frac{1 - 2v^2}{v}$

$\Rightarrow \int \frac{v}{1 - 2v^2} dv = \int \frac{dx}{x} + c_1$ 1

$$\Rightarrow \frac{1}{4}\int \frac{4v}{1-2v^2}\,dv = \log x + \log k$$

$$\Rightarrow -\frac{1}{4}\log\left(1-2v^2\right) = \log kx$$

$$\Rightarrow \log\left(1-2v^2\right)^{-\frac{1}{4}} = \log kx$$

$$\Rightarrow \frac{1}{\left(1-2v^2\right)^{\frac{1}{4}}} = kx$$

$$\Rightarrow 1 = k^4 . x^4\left(1-2v^2\right)$$

$$\Rightarrow 1 = c\ .\ x^2\left(x^2-2y^2\right)$$ 1

11. Let A, B, C, D are the events white balls draw

$$P(A) = \frac{5}{20} = \frac{1}{4},\ P(\overline{A}) = \frac{3}{4}$$

$$P(B) = \frac{5}{20} = \frac{1}{4},\ P(\overline{B}) = \frac{3}{4}$$

$$\therefore\ P(\overline{C}) = P(\overline{D}) = \frac{3}{4}$$

$$P(\overline{A}\,\overline{B}\,\overline{C}\,\overline{D}) = P(\overline{A})P(\overline{B})P(\overline{C})P(\overline{D})$$

$$= \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{81}{256}$$ 1

Required Prob. $= 1 - \frac{81}{256} = \frac{175}{256}$ 1

SECTION – C

12. L. H. S.

$$= \cot^{-1}\left[\frac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$$

$$= \cot^{-1}\left[\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right] \quad 1$$

L. H. S.

$$= \cot^{-1}\left[\frac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right] \quad 1$$

$$= \cot^{-1}\left[\frac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right] \quad 1$$

$$= \cot^{-1}\left[\cot\frac{x}{2}\right] = \frac{x}{2} = \text{R. H. S.} \quad 1$$

13. $f(1) = 1 - 1 = 0$

L. H. L. $= \lim_{x\to 1^-}(1-x) = \lim_{h\to 0} 1-(1-h) = 0$

R. H. L. $= \lim_{x\to 1^+}(x^2-1) = \lim_{h\to 0}(1+h)^2 - 1 = 0$

L. H. L. = R. H. L. = $f(1) = 0$

$\therefore f(x)$ is continuous at $x = -1$ 1

L. H. D. $= \lim_{x \to 1^-} \dfrac{f(x) - f(-1)}{x - 1}$

$= \lim_{x \to 1^-} \dfrac{(1 - x) - 0}{x - 1}$

$= \lim_{h \to 0} \dfrac{1 - (1 - h)}{1 - h - 1} = \lim_{h \to 0} \dfrac{h}{-h} = -1$ 1

R. H. D. $= \lim_{x \to 1^+} \dfrac{f(x) - f(1)}{x - 1}$

$= \lim_{x \to 1^+} \dfrac{x^2 - 1 - 0}{x - 1} = \lim_{h \to 0} \dfrac{(1 + h)^2 - 1}{1 + h - 1}$

$= \lim_{h \to 0} \dfrac{1 + h^2 + 2h - 1}{h} = \lim_{h \to 0} \dfrac{h(h + 2)}{h}$

$= 2$ 1

$\therefore f(x)$ is not derivable at $x = 1$ 1

14. $\dfrac{x^2}{a^2} - \dfrac{y^2}{b^2} = 1$ .... (i)

diff. w. r. t. x, we get

$\dfrac{2x}{a^2} - \dfrac{2y}{b^2}\dfrac{dy}{dx} = 0$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{b^2}{a^2} \cdot \dfrac{x}{y}$

at (x_0, y_0), $\frac{dy}{dx} = \frac{b^2}{a^2} \cdot \frac{x_0}{y_0}$ 1

$\therefore$ equation of tangent at (x_0, y_0)

$y - y_0 = \frac{b^2}{a^2} \frac{x_0}{y_0}(x - x_0)$ 1

$\Rightarrow \frac{yy_0 - y_0^2}{b^2} = \frac{xx_0 - x_0^2}{a^2}$

$\Rightarrow \frac{xx_0}{a^2} - \frac{yy_0}{b^2} = \frac{x_0^2}{a^2} - \frac{y_0^2}{b^2}$ .... (ii) 1

at (x_0, y_0) equation (i) is

$\frac{x_0^2}{a^2} - \frac{y_0^2}{b^2} = 1$ .... (iii)

from (ii) and (iii), we get

$\frac{xx_0}{a^2} - \frac{yy_0}{b^2} = 1$ 1

15. Let $X = \{0, 1, 2, 3, 4\}$ $\frac{1}{2}$

$P(X = 0) = \frac{{}^6C_4}{{}^{10}C_4} = \frac{1}{14}$ $\frac{1}{2}$

$P(X = 1) = \frac{{}^4C_1 \times {}^6C_3}{{}^{10}C_4} = \frac{8}{21}$ $\frac{1}{2}$

$P(X = 2) = \frac{{}^4C_2 \times {}^6C_2}{{}^{10}C_4} = \frac{6}{14}$ $\frac{1}{2}$

$$P(X = 3) = \frac{{}^4C_3 \times {}^6C_1}{{}^{10}C_4} = \frac{4}{35}$$ $\frac{1}{2}$

$$P(X = 4) = \frac{{}^4C_4}{{}^{10}C_4} = \frac{1}{210}$$ $\frac{1}{2}$

X	0	1	2	3	4
$P(X)$	$\frac{1}{14}$	$\frac{8}{21}$	$\frac{6}{14}$	$\frac{4}{35}$	$\frac{1}{210}$

1

16. $\vec{a} = \hat{i} + 2\hat{j} - \hat{k}$, $\vec{b} = 2\hat{i} + 3\hat{j} + \hat{k}$

Let $\vec{c} = c_1\hat{i} + c_2\hat{j} + c_3\hat{k}$ is perpendicular to $\vec{a}$ and $\vec{b}$ and $|\vec{c}| = 1$

$c_1 + 2c_2 - c_3 = 0$ .... (2) 1

$2c_1 + 3c_2 + c_3 = 0$ .... (2) 1

$$\frac{c_1}{2+3} = \frac{c_2}{-2-1} = \frac{c_3}{3-4} = k$$

$c_1 = 5k, c_2 = -3k, c_3 = -k$

and $c_1^2 + c_2^2 + c_3^2 = 1$ 1

$$\Rightarrow 25k^2 + 9k^2 + k^2 = 1 \Rightarrow k^2 = \frac{1}{35}$$

$$\Rightarrow k \pm \frac{1}{\sqrt{35}}$$

$\therefore \vec{c} = \pm\frac{1}{\sqrt{35}}\left[5\hat{i} - 3\hat{j} - \hat{k}\right]$ 1

SECTION – D

17. $2x - y + z = -1$

$-x + 2y - z = 4$

$x - y + 2z = -3$

$$\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$$ 1

$AX = B$

$\Rightarrow X = A^{-1}B$

$|A| = 2(4-1) + 1(-2+1) + 1\,(1-2)$

$= 6 - 1 - 1 = 4 \neq 0$ 1

$A_{11} = 3,\ A_{12} = 1,\ A_{13} = -1$

$A_{21} = 1,\ A_{22} = 3,\ A_{23} = 1$

$A_{31} = -1,\ A_{32} = 1,\ A_{33} = 3$

$$\therefore Adj\ A = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}' = \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix}$$

$$\therefore X = A^{-1}B = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 1 \\ -1 & 1 & 3 \end{bmatrix} = \begin{bmatrix} -1 \\ 4 \\ -3 \end{bmatrix}$$ 2

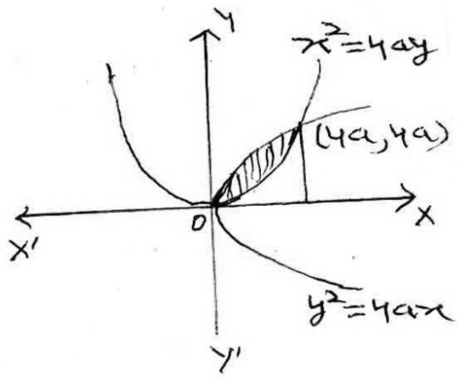
$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -3+4+3 \\ -1+12-3 \\ 1+4-9 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$ 1

$x = 1,\ y = 2,\ z = -1$ 1

18. $y^2 = 4ax$ ..... (i)

$x^2 = 4ay$ ..... (ii)

$$\left(\frac{x^2}{4a}\right)^2 = 4ax$$



2

$\Rightarrow x^4 = 64a^3x$

$\Rightarrow x\left(x^3 - 64a^3\right) = 0$

$\Rightarrow x = 0,\ 4a$

at $x = 0,\ y = 0$ and $x = 4a,\ y = 4a$

Required area $= \int_0^{4a} [y \text{ of curve (i)}]\, dx$ 1

$$- \int_0^{4a} [y \text{ of curve (ii)}]\, dx$$ 1

$$= \int_0^{4a} 2\sqrt{a}\sqrt{x}dx - \int_0^{4a} \frac{x^2}{4a}\ dx$$

$$= 2\sqrt{a}\left[\frac{2}{3}x^{3/2}\right]_0^{4a} - \frac{1}{4a}\left[\frac{x^3}{3}\right]_0^{4a}$$ 1

$$= \frac{4}{3}\sqrt{a}8a^{3/2} - \frac{1}{4a}.\frac{1}{3}.64.\ a^3$$

$$= \frac{32}{3}a^2 - \frac{16}{3}a^2 = \frac{16}{3}a^2 \text{ sq. units}$$ 1

OR

Let $I = \int_0^{\pi/2} \frac{x}{\sin x + \cos x}\, dx$ .... (i)

$$\therefore\ I = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\sin\left(\frac{\pi}{2} - x\right) + \cos\left(\frac{\pi}{2} - x\right)}\, dx$$

$\therefore\ \Rightarrow I = \int_0^{\pi/2} \frac{\frac{\pi}{2} - x}{\cos x + \sin x}\, dx$ .... (ii) 1

Adding (i) and (ii), we get

$$2I = \frac{\pi}{2}\int_0^{\pi/2}\frac{dx}{\sin x + \cos x}$$ 1

$$= \frac{\pi}{2}\frac{1}{\sqrt{2}}\int_0^{\pi/2}\frac{dx}{\frac{1}{\sqrt{2}}\sin x + \frac{1}{\sqrt{2}}\cos x}$$ 1

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\frac{dx}{\sin\frac{\pi}{4}\sin x + \cos\frac{\pi}{4}\cos x}$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\frac{dx}{\cos\left(\frac{\pi}{4} - x\right)} = \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\frac{dx}{\cos\left(x - \frac{\pi}{4}\right)}$$

$$= \frac{\pi}{2\sqrt{2}}\int_0^{\pi/2}\sec\left(x - \frac{\pi}{4}\right)dx$$

$$= \frac{\pi}{2\sqrt{2}}\left[\log\left\{\sec\left(x - \frac{\pi}{4}\right)\right\} + \tan\left(x - \frac{\pi}{4}\right)\right]_0^{\pi/2}$$ 1

$$= \frac{\pi}{2\sqrt{2}}\left[\log\left(\sec\frac{\pi}{4} + \tan\frac{\pi}{4}\right) - \log\left(\sec\frac{\pi}{4} - \tan\frac{\pi}{4}\right)\right]$$ 1

$$= \frac{\pi}{2\sqrt{2}}\left[\log\left(\sqrt{2}+1\right) - \log\left(\sqrt{2}-1\right)\right]$$

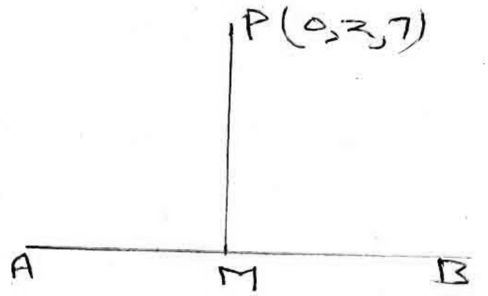
$$\Rightarrow I = \frac{\pi}{4\sqrt{2}}\log\left[\frac{\sqrt{2}+1}{\sqrt{2}-1}\times\frac{\sqrt{2}+1}{\sqrt{2}+1}\right] = \frac{\pi}{4\sqrt{2}}2\log\left(\sqrt{2}+1\right)$$

$$\therefore I = \frac{\pi}{2\sqrt{2}}\log\left(\sqrt{2}+1\right)$$ 1

19. $\frac{x+1}{-1} = \frac{y-1}{3} = \frac{z-3}{-2} = r$

$x = -r - 1,\ y = 3r + 1$

$z = -2r + 3$



1

Let

$\therefore$ M($-r - 1$, $3r + 1$, $-2r + 3$) 1

D.R's of line AB are $-1, 3, -2$ 1

and D.R's of PM are $-r - 1$, $3r - 1$, $-2r - 4$ 1

$AB \perp PM$

$\therefore 1\,(-r - 1) + 3\,(3r - 1) - 2(-2r - 4) = 0$

$\Rightarrow r + 1 + 9r - 3 + 4r + 8 = 0$

$\Rightarrow 14r = -6 \Rightarrow r = \frac{-3}{7}$ 1

$\therefore$ Foot of perpendicular $M \equiv \left(\frac{-4}{7}, \frac{-2}{7}, \frac{27}{7}\right)$ 1

OR

Equation of plane passing through (–2, 6, –6) is :

$a(x + 2) + b(y - 6) + c(z + 6) = 0$ .... (i) 1

Also passing through (–3, 10, –9) and (–5, 0, –6)

$\therefore -a + 4b - 3c = 0$ .... (ii) 1

$-3a - 6b - 0c = 0$ .... (iii) 1

$\frac{a}{0-18} = \frac{b}{9-0} = \frac{c}{6+12} \Rightarrow \frac{a}{2} = \frac{b}{-1} = \frac{c}{-2} = k$ 1

$\therefore 2k\,(x + 2) - k(y - 6) - 2k(z + 6) = 0$ 1

$\Rightarrow 2x - y - 2z - 2 = 0$ 1

20. For tabulation 2

For graphical representation 2

Minimum $z = 4$ 1

at $x = \frac{8}{7}$, $y = \frac{4}{7}$ 1

SET – D

SECTION – A

1. (i) $gof\,(x) = g\left(\dfrac{x+1}{x+2}\right)$

$$= \left(\frac{x+1}{x+2}\right)^2$$ **Ans. (A)** 1

(ii) Let $\sec^{-1}\dfrac{5}{3} = \theta \Rightarrow \sec\theta = \dfrac{5}{3}$

$$\therefore \cos\left(\sec^{-1}\frac{5}{3}\right) = \cos\theta$$

$$= \frac{1}{\sec\theta} = \frac{3}{5}$$ **Ans. (B)** 1

(iii) $2A + B = 2\begin{bmatrix} 2 & -1 \\ 4 & 2 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix}$

$$= \begin{bmatrix} 4 & -2 \\ 8 & 4 \end{bmatrix} + \begin{bmatrix} 4 & 3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 8 & 1 \\ 6 & 5 \end{bmatrix}$$ **Ans. (A)** 1

(iv) $8 - (3 - x) = 0$

$\Rightarrow 8 - 3 + x = 0 \Rightarrow x = -5$ **Ans. (D)** 1

(v) $\frac{d}{dx}\sqrt{1+\cot x} = \frac{1}{2}(1+\cot x)^{1/2} \cdot \frac{d}{dx}(1+\cot x)$

$= \frac{-\operatorname{cosec}^2 x}{2\sqrt{1+\cot x}}$ **Ans. (B)** 1

(vi) $f(x) = 2\cos x + \sqrt{3} \,.\, x$

$\therefore\ f'(x) = -2\sin x + \sqrt{3}$

For Maxima or Minima $f'(x) = 0$

$\Rightarrow 2\sin x = \sqrt{3} \Rightarrow \sin x = \frac{\sqrt{3}}{2}$

$x = \frac{\pi}{3}$ **Ans. (C)** 1

(vii) $x = a(\theta - \sin\theta),\ \ y = a[1 - \cos\theta]$

$\frac{dx}{d\theta} = a[1-\cos\theta],\ \frac{dy}{d\theta} = a\sin\theta$

$\therefore \frac{dy}{dx} = \frac{a\sin\theta}{a[1-\cos\theta]}$

at $\theta = \frac{\pi}{2}, \frac{dy}{dx} = \frac{\sin\frac{\pi}{2}}{1-\cos\frac{\pi}{2}} = \frac{1}{1-0} = 1$

slope of normal = -1 **Ans. (B)** 1

(viii) $\int \sin^2 \frac{x}{2}\, dx = \frac{1}{2}\int (1 - \cos x)\, dx$

$= \frac{1}{2}[x - \sin x] + c$ **Ans. (C)** 1

(ix) $\int \frac{x^2 - 1}{x^2 + 4}\, dx = \int \frac{x^2 + 4 - 5}{x^2 + 4}\, dx$

$= \int \left[1 - \frac{5}{x^2 + 4}\right] dx$

$= x - \frac{5}{2}\tan^{-1}\frac{x}{2} + c$ **Ans. (B)** 1

(x) Order = 2 **Ans. (C)** 1

(xi) $\sin^{-1}\frac{dy}{dx} = x$

$\Rightarrow \frac{dy}{dx} = \sin x$

Integrate w. r. t. x, we get

$y = -\cos x + c$ **Ans. (C)** 1

(xii) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\therefore\ 0.8 = 0.5 + 0.6 - P(A \cap B)$

$\therefore\ P(A \cap B) = 0.3$

$\therefore P(B/A) = \dfrac{P(A \cap B)}{P(A)} = \dfrac{0.3}{0.5} = \dfrac{3}{5}$ **Ans. (B)** 1

(xiii) $P(AB) = P(A)\ P(B/A)$

$= \dfrac{13}{52} \times \dfrac{13}{51} = \dfrac{13}{204}$ **Ans. (A)** 1

(xiv) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$\Rightarrow P(A \cup B) = P(A) + P(B) - P(A)\ P(B)$

$\Rightarrow 0.5 = 0.2 + P(B)\ [\ 1 - 0.2]$

$\Rightarrow 0.3 = 0.8\ P(B)$

$\Rightarrow P(B) = \dfrac{0.3}{0.8} = \dfrac{3}{8}$ **Ans. (C)** 1

(xv) $\vec{a}\ .\ \vec{b} = 0$

$\Rightarrow 2.1 + 1.\lambda + (-3)\ .\ 2 = 0$

$\Rightarrow 2 + \lambda - 6 = 0 \Rightarrow \lambda = 4$ **Ans. (D)** 1

(xvi) $3x + 1 = 6y - 2 = 1 - z$

$$\Rightarrow \frac{x+\frac{1}{3}}{\frac{1}{3}} = \frac{y-\frac{1}{3}}{\frac{1}{6}} = \frac{z-1}{-1}$$

$$\Rightarrow \frac{x+\frac{1}{3}}{2} = \frac{y-\frac{1}{3}}{1} = \frac{z-1}{-6}$$

$\therefore$ D. R's of given line are 2, 1, –6 **Ans. (A)** 1

SECTION – B

2. Let $x_1, x_2 \in Q$

$f(x_1) = f(x_2) \Rightarrow 3x_1 + 5 = 3x_2 + 5$

$\Rightarrow x_1 = x_2$ 1

$\therefore f(x)$ is one-one 1

3. Let $\tan^{-1} x = \alpha, \tan^{-1} y = \beta$

$\therefore\ x = \tan\alpha,\ y = \tan\beta$

Now $\tan(\alpha+\beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha . \tan\beta}$ 1

$$\Rightarrow \tan(\alpha+\beta) = \frac{x+y}{1-xy}$$

$$\Rightarrow (\alpha+\beta) = \tan^{-1}\frac{x+y}{1-xy}$$

$$\therefore \tan^{-1}x + \tan^{-1}y = \tan^{-1}\frac{x+y}{-xy}$$ 1

4. $A^2 = A.A$

$$= \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9+20 & -15-10 \\ -12-8 & 20+4 \end{bmatrix} = \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix}$$ 1

$$f(A) = A^2 - 5A - 14I$$

$$= \begin{bmatrix} 29 & -25 \\ -20 & 24 \end{bmatrix} - 5\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} - 14\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 29-15-14 & -25+25 \\ -20+20 & 24-10-14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$ 1

5. Area of triangle $= \frac{1}{2}\begin{vmatrix} 1 & -1 & 1 \\ 2 & 4 & 1 \\ -3 & 5 & 1 \end{vmatrix}$ 1

$$= \frac{1}{2}[1(4-5) + 1(2+3) + 1(10+12)]$$

$$= \frac{1}{2}[-1+5+22] = 13 \text{ sq. units}$$ 1

6. Let $y = (\sin x)^{\cos^{-1} x}$

taking log both side, we get

$\log y = \cos^{-1} x \,.\, \log \sin x$ 1

diff. w. r. t. x, we get

$$\frac{1}{y}\frac{dy}{dx} = \cos^{-1} x \,.\, \frac{1}{\sin x} \cos x + \log \sin \left(\frac{-1}{\sqrt{1-x^2}} \right)$$

$$\Rightarrow \frac{dy}{dx} = (\sin x)^{\cos^{-1} x} \left[\cos^{-1} x \,.\, \cot x - \frac{\log \sin x}{\sqrt{1-x^2}} \right]$$ 1

7. $x = a\,(1 + \cos \theta),\ y = a\,(\theta + \sin\theta)$

$\frac{dx}{d\theta} = -a \sin \theta, \frac{dy}{d\theta} = a(1 + \cos \theta)$ 1

$$\therefore \frac{dy}{dx} = \frac{a(1+\cos\theta)}{-a \sin \theta} = -\frac{2\cos^2 \frac{\theta}{2}}{2 \sin \frac{\theta}{2} \cos \frac{\theta}{2}} = -\cot \frac{\theta}{2}$$ 1

8. $\int \cot^{-1} x \; dx = \int \cot^{-1} x \,.\, 1 dx$

$$= \cot^{-1} x \,.\, \int 1 dx - \int \left[\frac{d}{dx} \cot^{-1} x . \int 1 dx \right] dx$$ 1

$$= x \,.\, \cot^{-1} x + \int \frac{1}{1+x^2} \,.\, x\, dx + c$$

$$= x \,.\, \cot^{-1} x + \frac{1}{2}\log\left(1+x^2\right) + c$$ 1

9. $\int \frac{dx}{32-2x^2} = \frac{1}{2}\int \frac{dx}{(4)^2 - x^2}$ 1

$$= \frac{1}{2} \times \frac{1}{2\times 4} \log \frac{4+x}{4-x} + c$$

$$= \frac{1}{16} \log \frac{4+x}{4-x} + c$$ 1

10. $\frac{dy}{dx} = \frac{x^2+y^2}{x^2+xy}$

Put $y = vx \Rightarrow \frac{dy}{dx} = v + x\frac{dv}{dx}$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{x^2 + v^2x^2}{x^2 + x\,.\,vx}$$

$$\Rightarrow v + x\frac{dv}{dx} = \frac{1+v^2}{1+v}$$

$$\Rightarrow x\frac{dv}{dx} = \frac{1+v^2}{1+v} - v$$

$$= \frac{1+v^2 - v - v^2}{1+v} = \frac{1-v}{1+v}$$

$$\Rightarrow \int \frac{1+v}{1-v} dv = \frac{dx}{x}$$ 1

$$\Rightarrow \int \left[\frac{2}{1-v} - 1 \right] dv = \int \frac{dx}{x} + c$$

$\Rightarrow -2 \log (1 - v) - v = \log x + c$

$$\Rightarrow -2 \log \left(1 - \frac{y}{x} \right) - \frac{y}{x} = \log x + c$$

$$\Rightarrow -2 \log \left(\frac{x-y}{x} \right) - \frac{y}{x} = \log x + c$$

$$\Rightarrow \log x - 2 \log y - \frac{y}{x} = c$$ 1

11. $P(A) = \frac{1}{2}, P(B) = \frac{1}{3}, P(C) = \frac{1}{4}$

$\therefore P(\overline{A}) = \frac{1}{2}, P(\overline{B}) = \frac{2}{3}, P(\overline{C}) = \frac{3}{4}$

Prob. that the problem could not be solved by any one of them $= P(\overline{A}\overline{B}\overline{C})$

$$= P(\overline{A})P(\overline{B})P(\overline{C})$$

$$= \frac{1}{2} \times \frac{2}{3} \times \frac{3}{4} = \frac{1}{4}$$ 1

$\therefore$ Prob. solved $= 1 - \frac{1}{4} = \frac{3}{4}$ 1

SECTION – C

12. L. H. S. $= \sin^{-1}\frac{14}{5} + \sin^{-1}\frac{5}{13} + \sin^{-1}\frac{16}{65}$

$$= \sin^{-1}\left[\frac{4}{5}\sqrt{1-\frac{25}{169}} + \frac{5}{13}\sqrt{1-\frac{16}{25}}\right] + \sin^{-1}\frac{16}{65} \quad 1$$

$$= \sin^{-1}\left(\frac{4}{5}\times\frac{12}{13} + \frac{5}{13}\times\frac{3}{5}\right) + \sin^{-1}\frac{16}{65} \quad 1$$

$$= \sin^{-1}\frac{63}{65} + \cos^{-1}\sqrt{1-\left(\frac{16}{65}\right)^2} \quad 1$$

$$= \sin^{-1}\frac{63}{65} + \cos^{-1}\frac{63}{65} = \frac{\pi}{2} = \text{R. H. S.} \quad 1$$

13. $f(0) = 2 + 0 = 2$

L. H. L. $= \lim_{x\to 0^-} (2-x) = \lim_{h\to 0} 2-(0-h) = 2$

R. H. L. $= \lim_{x\to 0^+} (2+x) = \lim_{h\to 0} 2+(0+h) = 2$

L. H. L. = R. H. L. = $f(0) = 2$

$\therefore f(x)$ is continuous at $x = 0$ 1

R. H. D. $= \lim_{x\to 0^+} \frac{f(x)-f(0)}{x-0}$

$$= \lim_{h\to 0} \frac{2+(0+h)-2}{(0+h)} = 1 \quad 1$$

L. H. D. $= \lim_{x \to 0^-} \frac{(2-x)-2}{x-0}$

$= \lim_{h \to 0} \frac{-(0-h)}{0-h} = -1$ 1

$\therefore$ $f(x)$ is not derivable at $x = 0$ 1

14. $x = y^2$ .... (i)

$xy = k$ .... (ii)

from (i) and (ii) $x = k^{2/3}, y = k^{1/3}$

$\therefore$ Point of intersection is $\left(k^{2/3}, k^{1/3}\right)$

diff. (i) w. r. t. x, we get

$1 = 2y\frac{dy}{dx} \Rightarrow \frac{dy}{dx} = \frac{1}{2y}$

At $\left(k^{2/3}, k^{1/3}\right)$, $\frac{dy}{dx} = \frac{1}{2k^{1/3}}$

$\therefore$ m_1 = slope of tangent of (i) $= \frac{1}{2k^{1/3}}$ 1

diff. (ii) w. r. t. x, we get

$x\frac{dy}{dx} + y \cdot 1 = 0 \Rightarrow \frac{dy}{dx} = -\frac{y}{x}$

at $\left(k^{2/3}, k^{1/3}\right)$, $\frac{dy}{dx} = -\frac{k^{1/3}}{k^{2/3}} = -\frac{1}{k^{1/3}}$

$\therefore$ m_2 = slope of tangent of (ii) $= -\frac{1}{k^{1/3}}$ 1 curve (i) is perpendicular curve (ii),

$\therefore m_1.m_2 = -1$ 1

$$\frac{1}{2.k^{1/3}}.\left(-\frac{1}{k^{1/3}}\right) = -1$$

$\Rightarrow k^{2/3} = \frac{1}{2} \Rightarrow k^2 = \frac{1}{8} \Rightarrow 8k^2 = 1$ 1

15. Let $X = 0, 1, 2, 3$

$P(S) = P = \frac{4}{7}, P(F) = q = \frac{3}{7}$ 1

$P(X = 0) = P(FFF) = P(F)P(F)P(F)$

$= \frac{3}{7}\times\frac{3}{7}\times\frac{3}{7} = \frac{27}{343}$ $\frac{1}{2}$

$P(X = 1) = P(SFF \text{ or } FSF \text{ or } FFS)$

$= \frac{4}{7}\times\frac{3}{7}\times\frac{3}{7}+\frac{3}{7}\times\frac{4}{7}\times\frac{3}{7}+\frac{3}{7}\times\frac{3}{7}\times\frac{4}{7} = \frac{108}{343}$

$\frac{1}{2}$

$P(X = 2) = P(SSF \text{ or } SFS \text{ or } FSS)$

$= \frac{4}{7}\times\frac{4}{7}\times\frac{3}{7}+\frac{4}{7}\times\frac{3}{7}\times\frac{4}{7}+\frac{3}{7}\times\frac{4}{7}\times\frac{4}{7} = \frac{144}{343}$ $\frac{1}{2}$

$P(X = 3) = P(SSS) = \frac{4}{7}\times\frac{4}{7}\times\frac{4}{7} = \frac{64}{343}$ $\frac{1}{2}$

X	0	1	2	3
$P(X)$	$\frac{27}{343}$	$\frac{108}{343}$	$\frac{144}{343}$	$\frac{64}{343}$

1

16. Let $A \equiv (1, 2, 4)$, $B \equiv (3, 1, -2)$, $C \equiv (4, 3, 1)$

$\vec{a} = \hat{i} + 2\hat{j} + 4\hat{k}$, $\vec{b} = 3\hat{i} + \hat{j} - 2\hat{k}$ and $\vec{c} = 4\hat{i} + 3\hat{j} + \hat{k}$

$\overrightarrow{BC} = \vec{c} - \vec{b} = \hat{i} + 2\hat{j} + 3\hat{k}$ 1

$\overrightarrow{BA} = \vec{a} - \vec{b} = -2\hat{i} + \hat{j} + 6\hat{k}$

$= \overrightarrow{BC} \times \overrightarrow{BA} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 2 & 3 \\ -2 & 1 & 6 \end{vmatrix}$ 1

$= \hat{i}(12 - 3) - \hat{j}(6 + 6) + \hat{k}(1 + 4)$

$= 9\hat{i} - 12\hat{j} + 5\hat{k}$ 1

$\therefore$ Area of $\Delta ABC = \frac{1}{2} | \overrightarrow{BC} \times \overrightarrow{BA} |$

$= \frac{1}{2}\sqrt{81 + 144 + 25}$

$= \frac{1}{2}\sqrt{250} = \frac{5}{2}\sqrt{10}$ sq. units 1

SECTION – D

17. $x + y + z = 6$

$y + 3z = 11$

$x - 2y + z = 0$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$ 1

$AX = B$

$\Rightarrow X = A^{-1}B$

$|A| = 1(1 + 6) - 1(0 - 3) + 1(0 - 1)$

$= 7 + 3 - 1 = 9 \neq 0$ 1

$A_{11} = 7,\ A_{12} = 3,\ A_{13} = -1$

$A_{21} = -3,\ A_{22} = 0,\ A_{23} = 3$

$A_{31} = 2,\ A_{32} = -3,\ A_{33} = 1$

$$\therefore Adj\ A = \begin{bmatrix} 7 & 3 & -1 \\ -3 & 0 & 3 \\ 2 & -3 & 1 \end{bmatrix}' = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$ 2

$$\therefore X = A^{-1}B = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

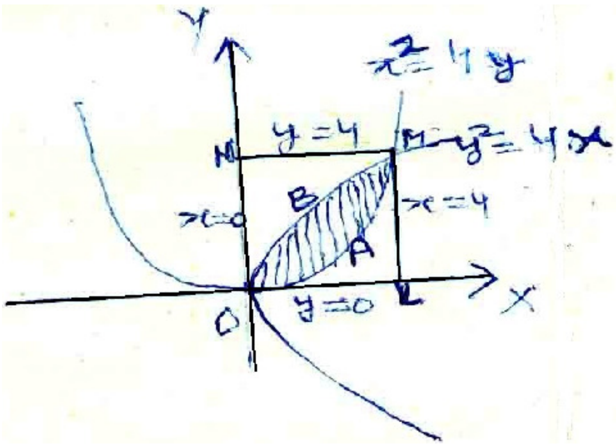
$$\therefore \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 42 - 33 + 0 \\ 18 - 0 + 0 \\ -6 + 33 + 0 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$ 1

$x = 1,\ y = 2,\ z = 3$ 1

18. $y^2 = 4x$ ..... (i)

$x^2 = 4y$ ..... (ii)

$$\left(\frac{x^2}{4}\right)^2 = ax$$



$\Rightarrow x^4 - 64x = 0$

$\Rightarrow x(x^3 - 64) = 0$

$\Rightarrow x = 0,\ x = 4$

when $x = 0,\ y = 0$ and when $x = 4,\ y = 4$

Area of region $OAMBO = \int_0^4 \left[2\sqrt{x} - \frac{x^2}{4}\right] dx$

$$= \left[2 \cdot \frac{2}{3} x^{3/2} - \frac{x^3}{12}\right]_0^4$$

$$= \frac{4}{3} \times 8 - \frac{64}{12} = \frac{32}{3} - \frac{16}{3} = \frac{16}{3} \text{ sq. units}$$

2

Area of region $OLMAO = \int_0^4 \frac{x^2}{4}\, dx$

$$= \left[\frac{x^3}{12}\right]_0^4 = \frac{64}{12} = \frac{16}{3} \text{ sq. units}$$

2

Area of region $OBMNO = \int_0^4 \frac{y^2}{4}\, dy$

$$= \left[\frac{y^3}{12}\right]_0^4 = \frac{64}{12} = \frac{16}{3} \text{ sq. units}$$

2

OR

Let $I = \int_0^{\pi/2} \frac{\sin^2 x}{1 + \sin x \cos x}\, dx$ .... (i)

$$\therefore\ I = \int_0^{\pi/2} \frac{\sin^2\left(\frac{\pi}{2} - x\right) dx}{1 + \sin\left(\frac{\pi}{2} - x\right)\cos\left(\frac{\pi}{2} - x\right)}$$

$\therefore\ I = \int_0^{\pi/2} \frac{\cos^2 x}{1 + \cos x \sin x}\, dx$ .... (ii) 1

Adding (i) and (ii), we get

$$2I = \int_0^{\pi/2} \frac{\sin^2 x + \cos^2 x}{1 + \sin x \cos x} dx = \int_0^{\pi/2} \frac{dx}{1 + \sin x \cos x} \qquad 1$$

$$= \int_0^{\pi/2} \frac{\sec^2 x dx}{\sec^2 x + \tan x} \qquad 1$$

$$\therefore 2I = \int_0^{\pi/2} \frac{\sec^2 x}{1 + \tan^2 x + \tan x} dx$$

Put $\tan x = t$
$\sec^2 x \, dx = dt$

$$= \int_0^{\infty} \frac{dt}{1 + t + t^2} = \int_0^{\infty} \frac{dt}{\left(t + \frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2} \qquad 1$$

$$= \frac{2}{\sqrt{3}} \left[\tan^{-1} \frac{t + \frac{1}{2}}{\frac{\sqrt{3}}{2}} \right]_0^{\infty} = \frac{2}{\sqrt{3}} . \left[\tan^{-1} \infty - \tan^{-1} \frac{1}{\sqrt{3}} \right] \qquad 1$$

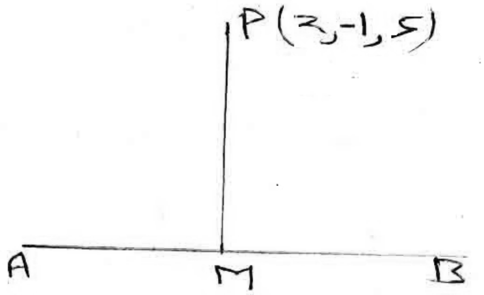
$$2I = \frac{2}{\sqrt{3}} \left[\frac{\pi}{2} - \frac{\pi}{6} \right] = \frac{2}{\sqrt{3}} \frac{2\pi}{6}$$

$$\therefore I = \frac{\pi}{3\sqrt{3}} \qquad 1$$

19. Line AB is

$$\frac{x - 11}{10} = \frac{y + 2}{-4} = \frac{z + 8}{-11} = r$$

General point on AB



1

Say M(10r + 11, –4r – 2, –11r – 8)

D.R's of AB are 10, –4, –11 and 1

D.R's of PM are 10r + 9, –4r – 1, –11r – 13 1

AB is perpendicular to PM

$\therefore 10(10r+9) - 4(-4r-1) - 11(-11r-13) = 0$ 1

$\Rightarrow 100r + 16r + 121r = -90 - 4 - 143$

$\Rightarrow 237r = -237 \Rightarrow r = -1$ 1

$\therefore$ Foot perpendicular $\equiv$ (1, 2, 3) 1

OR

Equation of plane passing through (–2, 6, –6) is :

$a(x+2) + b(y-6) + c(z+6) = 0$ .... (i)

Plane (i) passing (–3, 10, –9) and (–5, 0, –6)

$-a + 4b - 3c = 0$ .... (ii) 1

$-3a - 6b + 0c = 0$ .... (iii) 1

$\frac{a}{0-18} = \frac{b}{-(0-9)} = \frac{c}{6+12}$ 1

$\Rightarrow \frac{a}{2} = \frac{b}{-1} = \frac{c}{-2} = k$ 1

$\therefore$ From (i), we get

$\Rightarrow 2k\,(x + 2) - k(y - 6) - 2k(z + 6) = 0$ 1

$\Rightarrow 2x - y - 2z - 2 = 0$ 1

20. For tabulation 2

For graphical representation 2

Minimum $z = 27$ 1

at $x = 3,\ y = 4$ 1